

IMPROVED SPIRAL GEOMETRY FOR HIGH SPEED RAIL AND PREDICTED VEHICLE RESPONSE

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ABSTRACT

This paper compares two railroad curve transition spiral shapes with respect to their dynamic performance. The shapes compared are the traditional linear shape and an improved shape intended to provide optimal dynamic performance. The improved shape is attractive because it gives much better dynamic performance and also because it flows from an improved way of thinking about spirals.

The paper explains the improved spiral design method and the improved spiral shape to be evaluated. It then compares results from simulation of the operation of a rail vehicle over an improved spiral and over the corresponding traditional spiral. The simulations were done using the NUCARS simulation program and a model of the Amtrak ACELA passenger car. The simulated motion of the vehicle over the traditional spiral exhibits undesirable fluctuations of two types with different strengths and distinct causes.

The weaker fluctuations are the result of impacts of wheel flanges against the outside rail part way through the spiral. These impacts might be softened or eliminated by use of a truck design that could “steer” more effectively than the design that was simulated (or by making spirals much longer). They are not much affected by the kind of spiral geometry employed.

The stronger fluctuations are caused by the shape of the traditional spiral and particularly by the abruptness with which superelevation ramp angle changes at each end. The improved spiral geometry eliminates these fluctuations almost completely. It will thereby improve ride comfort and may reduce cost for maintaining spiral track alignment.

THE ROLE OF THE SPIRAL AND ITS TRADITIONAL FORM

Between a section of straight track with zero bank angle and a section of curved track with curvature and bank angle both constant and non-zero it is necessary to have a transition section in which bank angle varies with distance and that matches the adjacent bank angle at each end. Normally the curvature of such a transition section also varies with distance and matches the curvature of the adjacent section at each end. Such a transition is referred to as a spiral. In the form of railroad track spiral most widely used in the United States, the bank angle and curvature both vary linearly with distance along the transition. A spiral in which the curvature varies linearly with distance is sometimes referred to as a clothoid spiral and is referred to in this paper as a linear spiral.

SOME TERMINOLOGY AND CONCEPTS

The bank angle of the track will generally be referred to here as the **roll angle**.

The **curvature** of the track is equal to the derivative of the local compass direction of the track (in radians) with respect to distance along the track.

Considering two neighboring sections of track, each of which has constant curvature, the **offset** between the two adjacent track sections is the smallest distance between extensions of the sections that maintain their respective fixed curvatures. Two examples of offset between a section of tangent track and a curve are illustrated in Figure 1.

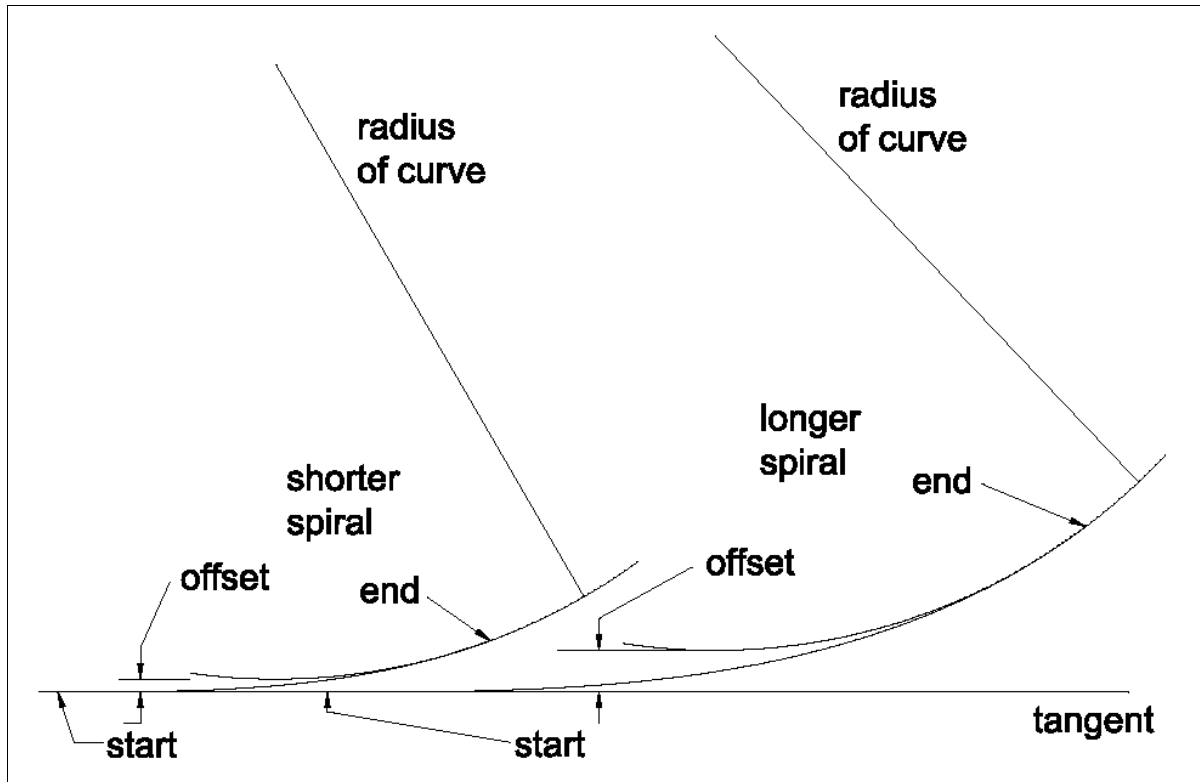


FIGURE 1 Spirals from tangent track to two curves; both curves have the same radius; when spiral length is increased, offset from the tangent is also increased.

The two spirals in Figure 1 illustrate a significant feature of spiral geometry, namely that, of two spirals that accomplish a given change of curvature, the longer spiral with the lower rate of change of curvature will have the larger offset.

In the past when many existing rail routes were first laid out, surveying and track lining were done by hand, and curved track was more expensive to build and maintain than straight track. Track tended to be a bit rough then, and not too much attention was paid to dynamic performance of spirals. As a result, many existing spirals are shorter than would be desired. Because of advances in technology for surveying and for track lining, the old preference for short spirals should be replaced by a preference for long spirals. Improvement of spiral shape (the subject of this paper) and lengthening of spirals contribute in complementary ways to improvement of spiral dynamic performance. Improvement of spiral shape is not a substitute for adequate spiral length.

It is normal in railroad curve and spiral design to impose a particular relationship between track roll angle and track curvature. The idea of the relationship is to bank the track so that the acceleration of gravity will have a component in the plane of the track and to adjust the roll angle so that that component provides the centripetal acceleration inherent in travel along the curve at a given “design” speed. This is illustrated in Figure 2.

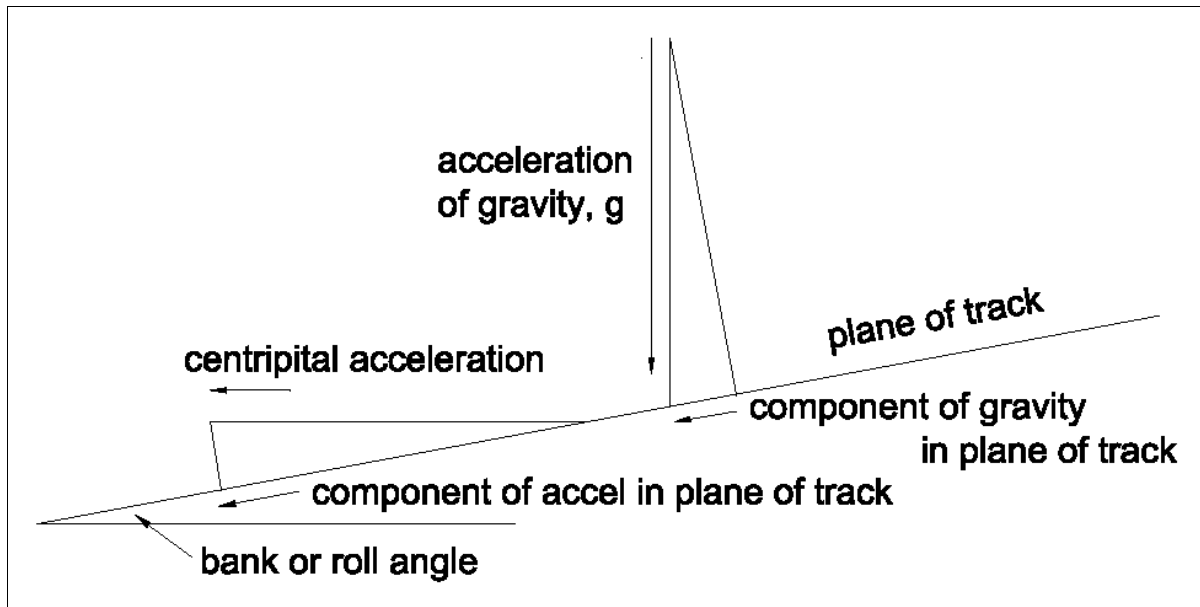


FIGURE 2 How bank angle affects components of gravitational and centripetal acceleration in plane of track.

The mathematical expression of this relationship is

$$(v_d)^2 db/ds \cos(r(s)) = g \sin(r(s)). \quad (1)$$

Here v_d is the design speed of the curve, b is the compass bearing angle of the track (in radians) at distance, s , along the track, db/ds is the derivative of bearing angle with respect to distance along the track (which is by definition the curvature of the track), $r(s)$ is the roll angle of the track at the distance s , \cos and \sin are the ordinary trigonometric functions, and g is the acceleration of gravity. This relation can be used to specify curvature as a function of distance along a spiral if roll angle has already been determined as a function of distance or vice-versa. Solving equation (1) for the curvature we have

$$db/ds = (g / (v_d)^2) \tan(r(s)). \quad (2)$$

It is helpful to note that in conventional railroad practice the roll angle of the track is limited to about 0.12 radians and that for roll angles whose magnitudes do not exceed that value the difference between $r(s)$ and $\tan(r(s))$ does not exceed 0.5 %. Thus, for conceptual understanding the track roll angle can be considered proportional to the track curvature and consequently the roll acceleration (about the longitudinal axis) of a truck traversing a spiral can be considered proportional to the second derivative of the track curvature.

THE DEFICIENCY OF THE LINEAR SPIRAL

When the traditional linear spiral is considered from the point of view of how it affects rail vehicle motions, it can be seen to have three deficiencies. Two of them are easy to understand from simple physics. The third relates to the complex behavior of railroad trucks.

The first deficiency of the linear spiral is that when a vehicle encounters either end of the linear ramp of superelevation along the spiral, the spiral tries conceptually to have the vehicle roll velocity change abruptly from zero to a non-zero value. Such a change involves a large angular acceleration and corresponding large forces. In practice, the change of roll velocity is spread over some time because of the combination of the bending stiffness of the rails, the roll compliance of the vehicle suspension, and the fact that the two trucks of the vehicle encounter the end of the spiral at different times. However, there is still some lurching that can be noticeable to passengers, and there are systematic transverse reaction forces applied by the vehicle trucks to the track structure. The foregoing deficiency has been recognized for a long time. (See for example the 1963 report by AAR (1) and the survey by Kufver (2)).

The second deficiency is that the linear spiral tries to roll vehicles about a longitudinal roll axis in the plane of the track. The forces required to accomplish a given amount of roll can be reduced if instead the vehicle is rolled about an axis whose height is close to the heights of the centers of mass of the vehicles. This opportunity for improvement has been dealt with by Presle and Hasslinger (3) who reference a 1968 paper by W. von Donges (4).

The third deficiency of the linear spiral has to do with the way that vehicle wheels and trucks “steer” within the gauge tolerance and how they respond to changes in curvature. With the linear spiral the derivative of the curvature with respect to distance is discontinuous at each end of the spiral. Exploratory simulations have indicated that a discontinuity in the derivative of track curvature can cause undesirable episodes of “hunting” with some truck designs.

PREVIOUS SUGGESTIONS OF BETTER SHAPES FOR SPIRALS

Prior suggestions of spiral shapes with more favorable dynamic characteristics have been reviewed in a very helpful study by B. Kufver (2). The prior suggestions all recognize that it is dynamically inappropriate to try conceptually to induce infinitely large vehicle roll accelerations at the ends of spirals. Accordingly, prior suggestions for spiral improvement have curvatures whose second derivatives with respect to distance are finite throughout the spiral. Figures (3) and (4) illustrate two previously suggested improved spiral shapes from among those reviewed by Kufver (2):

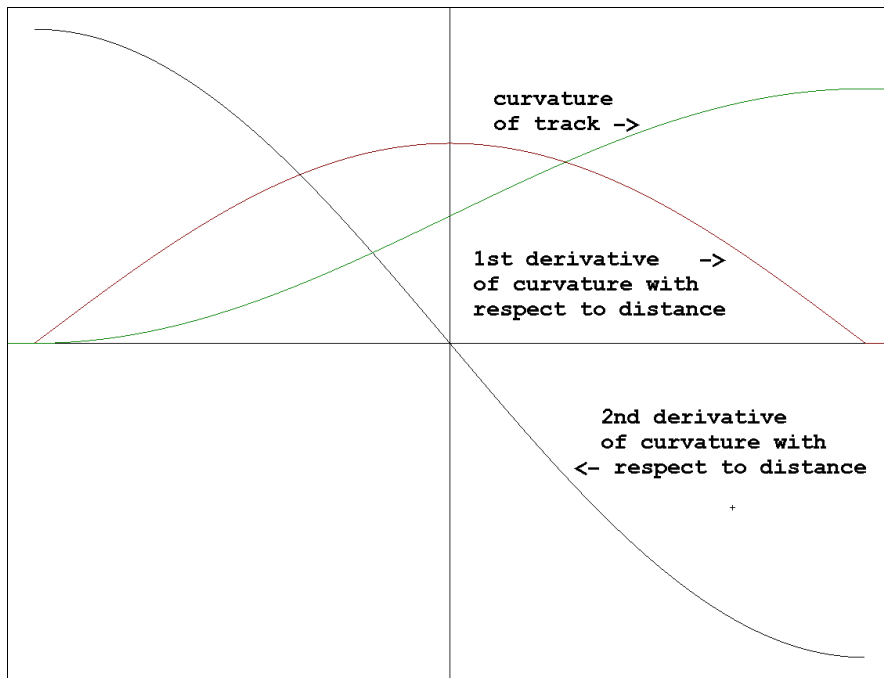


FIGURE 3 2nd derivative of curvature has form of a half period of a cosine (Vojacec, 1868). (Scales are arbitrary to illustrate shapes.)

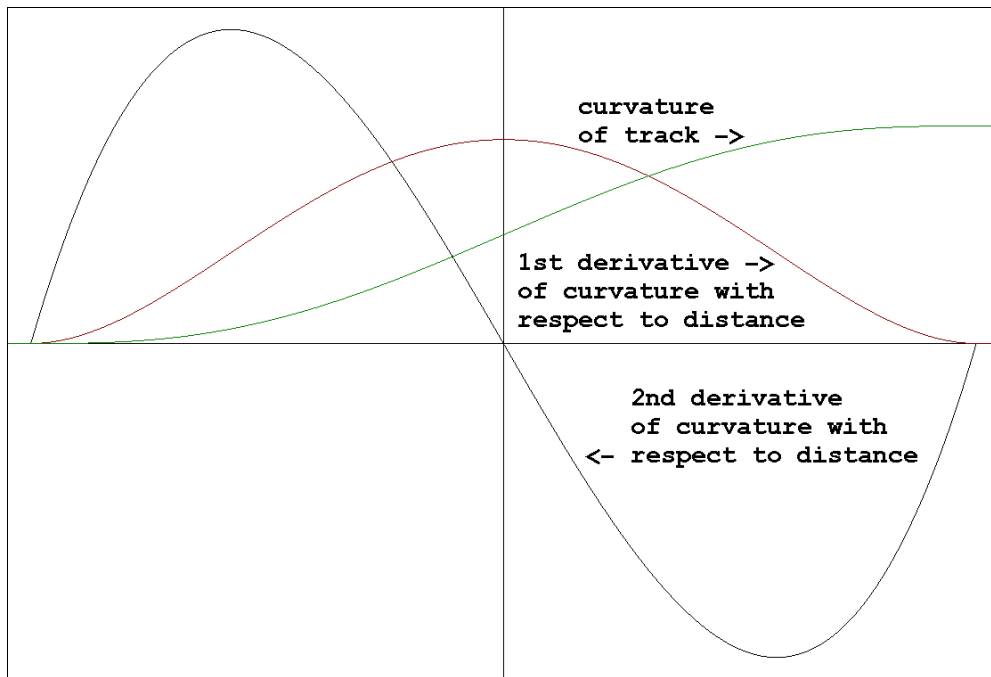


FIGURE 4 2nd derivative of curvature has form of a simple cubic with a zero in center of spiral and at each end (Watorek, 1907).

Of the previous suggestions for improved spiral shapes, the two that are most attractive from the perspective of the present paper can be called the cubic and sinusoidal spirals. The cubic spiral, illustrated above, has curvature whose second derivative is a cubic polynomial with a first order zero in the center of the spiral and at each end. The sinusoidal

spiral closely resembles the cubic spiral. It is called sinusoidal because the second derivative of its curvature is proportional to a full cycle of the sine function with value zero in the middle of the spiral and at each end. These two spiral shapes have third as well as second derivatives of curvature that are finite throughout the spiral. In other words, the roll motions that they induce have angular jerk which while not continuous at the end points of the spiral is at least finite throughout the spiral.

Another feature shared by the prior suggestions for spiral improvement is that they begin with the choice of a spiral shape.

A CONCEPTUAL ADVANCE FOR DESIGN OF SPIRALS

The spiral design method that is employed in the present paper (Klauder, (5) (6)) differs from prior suggestions for spiral improvement in that it begins not with choice of spiral shape but rather with choice of the manner in which vehicle trucks should roll about the longitudinal axis as they traverse a spiral. Quantitative differences due to this difference in approach can be small. However, the new approach is better conceptually because it prompts designers to recognize that one primary purpose of a spiral is to get vehicles to change bank angle by rolling in a graceful manner with minimum applications of force and with minimum unnecessary fluctuations of force and acceleration. Once a choice has been made of a functional form for the roll (i.e., bank) angle, $r(s)$, as a function of distance along a spiral, equation (2) above dictates the track curvature as a function of distance along the spiral. The curvature is then integrated with respect to distance to obtain the compass bearing angle of the track as a function of distance along the spiral, and the sine and cosine of the bearing angle are integrated with respect to distance to obtain the Cartesian coordinates of points along the spiral. These integrations are carried out numerically and can be performed quite quickly by programs that run on personal computers.

A TRACK ROLL FUNCTION WITH GENTLE ACCELERATION

A spiral shape can be characterized by its curvature as a function of distance along the spiral. Similarly, a spiral roll function can be characterized by $r(s)$, the roll angle as a function of distance along the track. However, for physical understanding of what is going on, it is necessary to look first not at $r(s)$ but rather at $d^2r(s)/ds^2$, its second derivative with respect to distance. For a vehicle traveling at a constant speed v , the corresponding angular acceleration of a truck about the longitudinal roll axis is related to $d^2r(s)/ds^2$ by the equation

$$d^2r(s(t))/dt^2 = v^2 d^2r(s)/ds^2 \quad (3) .$$

For simplicity, we will ignore the factor of speed squared and apply the name roll acceleration to $d^2r(s)/ds^2$. The reason for focusing attention on the roll acceleration rather than on the roll itself is that the imposed roll acceleration will determine the forces that accompany the roll motion and the accelerations felt by passengers. Thus, one main idea in the method is to devise a roll acceleration function that accomplishes a specified total roll, that is gentle and smooth, and that does not cause the track warp in the middle of the spiral to have too large a value. If curve offset is generous so that spirals will be long, then track warp will not be a problem, and roll acceleration can be made gentle and smooth by being spread out over the length of the spiral.

The roll function studied in the present paper is denoted as order {2,1}. It is illustrated in Figure (5).

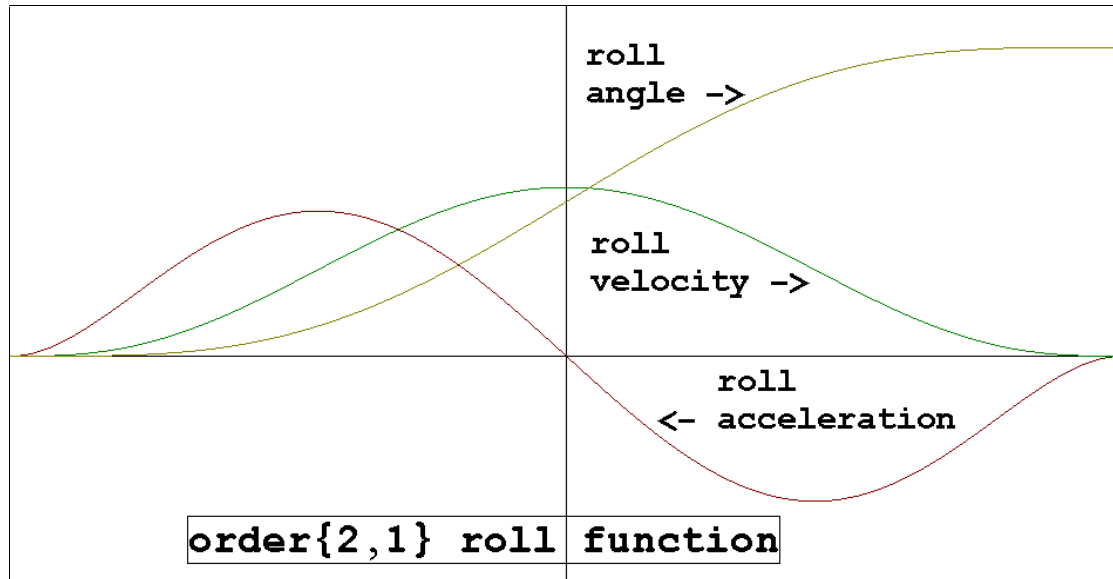


FIGURE 5 Order {2,1} roll function; roll acceleration has simple zero at mid point and 2nd order zero at each end.

The order {2,1} roll function and each of its derivatives is given by a single function that applies throughout the length of the spiral. The order {2,1} designation is used to indicate that the roll acceleration has a 2nd order zero at each end of the spiral and a 1st order zero at the midpoint of the spiral. Let the length of the spiral be 2a, denote distance along the spiral by s with s = 0 at the center of the spiral, and denote the change in the roll angle of the track over the length of the spiral by “roll_change”. Then the formula for the roll acceleration (meaning the 2nd derivative with respect to distance) is

$$d^2r/ds^2 = \frac{-105 \text{ roll_change } (a+s)^2 (a-s)^2 s}{16 a^7} \quad (4)$$

It may be observed that the order {2,1} roll function illustrated in Figure 5 is similar to the “cubic” spiral shape function reviewed by Kufver and credited to Watorek as illustrated in Figure 4. The differences are that the order {2,1} roll has angular jerk that is zero at each end of the spiral and that its immediate use is to define roll rather than curvature.

A number of other functional forms that can be used in roll acceleration functions for spirals are illustrated in reference (6).

RAISING THE HEIGHT OF THE ROLL AXIS

As emphasized by Presle and Hasslinger (3), the forces required to accomplish vehicle roll during spiral traversal can be substantially reduced by raising the vehicle roll axis above the plane of the track to a height that is approximately the height of the vehicle center of gravity.

The path of the spiral as derived above from the roll function is taken as the path of the roll axis, and the lateral displacement to the path of the track is obtained by simple

geometry from the roll axis height and the roll angle as a function of distance. By that same geometry, applicable with the roll axis raised, a discontinuity in the angular jerk causes a discontinuity in the derivative of the track curvature. The above-mentioned propensity of some trucks to begin hunting when they encounter discontinuity in the derivative of track curvature is the reason that reference (6) advocates roll functions with angular jerk zero at each end and continuous throughout.

Presle and Hasslinger (3) reported on a service test of an improved spiral shape similar to that studied in this paper applied in combination with a raising of the height of the roll axis. Prior to applying improved spiral geometry, the alignment at each end of a spiral degraded progressively over time because of the lateral reaction forces caused by the traditional linear spiral shape. They found that, with the improved spiral geometry including raised roll axis height, the lateral reaction forces applied by vehicles to the track at the ends of spirals were reduced sufficiently that the track alignment no longer suffered from the progressive deterioration that had been experienced previously.

If the focus is on improving passenger ride comfort, it may be desirable to have the vehicle roll axis at about the height of the shoulders of a seated passenger.

Raising the height of the axis about which track roll takes place has the additional benefit of causing an increase of spiral length and of thereby reducing the warp of the track.

CALCULATING THE GEOMETRY OF THE SPIRAL

For design purposes, the offset of the spiral and the curvature at each end of the spiral will have been specified. For a given spiral shape, the bounding curvatures and the offset will determine the geometry of the spiral. The roll function we will examine is defined by equation (4) above. The curvature is known at each end of the spiral so that by solving equation (2) above for roll angle as a function of curvature we know the roll angle at each end of the spiral and thus the factor roll_change in equation (4).

The calculation procedure set forth in reference (6) is composed of eight steps as follows:

- [1] - Choose an estimated value for the spiral half-length parameter, a , in equation (4);
- [2] - Obtain the formula for the roll velocity as a function of distance along the spiral by integrating equation (4) symbolically and setting the integration constant to zero;
- [3] - Obtain the formula for the roll angle by integrating symbolically a second time and taking the initial roll angle as the integration constant;
- [4] - Put the resulting formula for the roll angle into equation (2) and integrate it numerically to obtain the compass-bearing angle as a function of distance along the spiral;
- [5] - Numerically integrate the sine and cosine of the compass-bearing angle to obtain the Cartesian coordinates of points along the path of the spiral;
- [6] - Treat the path so obtained as the path of the roll axis, treat each roll axis point as being at the stipulated height above the track, and determine the coordinates of the corresponding track spiral point from the roll angle at that point and the roll axis height;
- [7] - Using the shape of the track spiral that results from the current estimate for the half-length, a , apply trigonometry to find the offset of the spiral;
- [8] - Repeat steps 1 through 7 above with improved estimates for the spiral length until the spiral that has the prescribed offset is obtained.

The data presented herein were obtained with the help of computer programs that implement the forgoing procedure.

SIMULATING VEHICLE RESPONSE TO A SPIRAL GEOMETRY

Our comparison of vehicle responses to two competing spiral shapes is based on simulation using the NUCARS rail vehicle motion simulation program developed by the Association of American Railroads. This program is available commercially from the Transportation Technology Center, Inc. (7). The NUCARS program reads input data from a file defining the geometry of the track and from a file defining mechanical characteristics of the vehicle to be simulated. A file of vehicle mechanical characteristics is referred to as a NUCARS vehicle model. The results presented herein are based on a NUCARS model of an ACELA coach that was developed for Amtrak by TTCI and that is used in this work by courtesy of Amtrak.

GEOMETRIES WHOSE VEHICLE RESPONSES ARE COMPARED

We compare two spiral shapes that can connect tangent track to a 1.0 degree curve that is offset from the tangent track by 0.55m (1.818 ft). The traditional linear spiral for this transition has length 152.4 m (500 ft). The curve is superelevated by 14.35 cm (5.65 in) for a balancing speed of 144.8 km/h (90 mph), and vehicles are simulated traveling at 183.5 km/h (114 mph) so that the so-called cant deficiency is 7.6 cm (3 in).

The offset is generous enough so that track warp will not be an issue. Accordingly, we can use an improved spiral shape with low values of roll angular acceleration and angular jerk. Of the shapes offered in reference (6), one of the ones with low values of roll angular acceleration and angular jerk is the order {2,1} shape illustrated in Figure 5, and that is the shape whose vehicle responses will be compared here with vehicle responses to the traditional linear spiral. Some comparisons not reported here suggest that the roll axis height that will give the best results will be about 2.44 m (8.0 ft). That is therefore the value used for the roll axis height of the order {2,1} spiral in the following comparison. The geometries of the traditional and order {2,1} spirals being compared are illustrated in Figure 6.

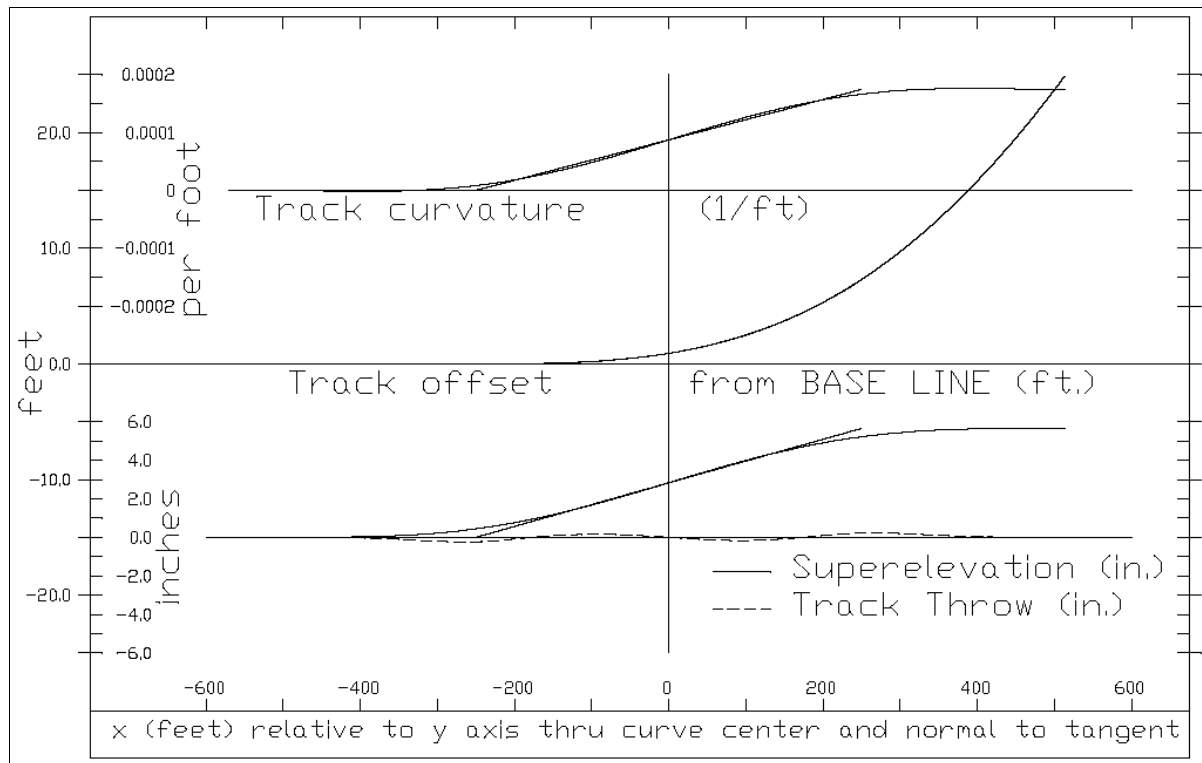


FIGURE 6 Plots of curvature, alignment, and superelevation for an order {2,1} spiral with 2.44 m (8 ft) roll axis height and for a corresponding traditional linear spiral. Both spirals connect tangent track to a 1.0 deg curve elevated for balance at 90 mph. (The two alignments shown in the central part of the plot are too close together to be distinguished at this scale. The displacement from one to the other is indicated by the track throw.) The lengths of the traditional and improved spirals illustrated are 152.4 m (500 ft) and 313.6 m (1028.8 ft) respectively.

VEHICLE DYNAMIC RESPONSE FEATURES OF INTEREST

The main simulated vehicle responses we present are car body lateral acceleration at mid car 2.13 m (7 ft) above top of rail (i.e., approximately at shoulder height of seated passengers) and net lateral force applied by all four wheels of each truck to the track. Following remarks on these responses we present results that illustrate the “steering” behavior of the vehicle’s two trucks as they traverse the spiral.

The strength of fluctuations of mid-car lateral acceleration and of the lateral forces applied to the track by the two trucks can be considered respectively as measures of disturbance to passengers and of disturbance to track alignment. It is desirable to minimize those fluctuations. As will be seen in the figures that follow, compared to the traditional linear spiral, the order {2,1} spiral shape with 2.44 m (8 ft) roll axis height achieves a substantial reduction in each of these measures of disturbance.

The predicted car body lateral accelerations at mid car at height 2.13 m (7.0 ft) above the track for the linear and order {2,1} spirals are shown in Figure 7. Simulated values in this and the following figures have x coordinates that indicate where the center of the coach was relative to the mid-point of the spiral when the value occurred.

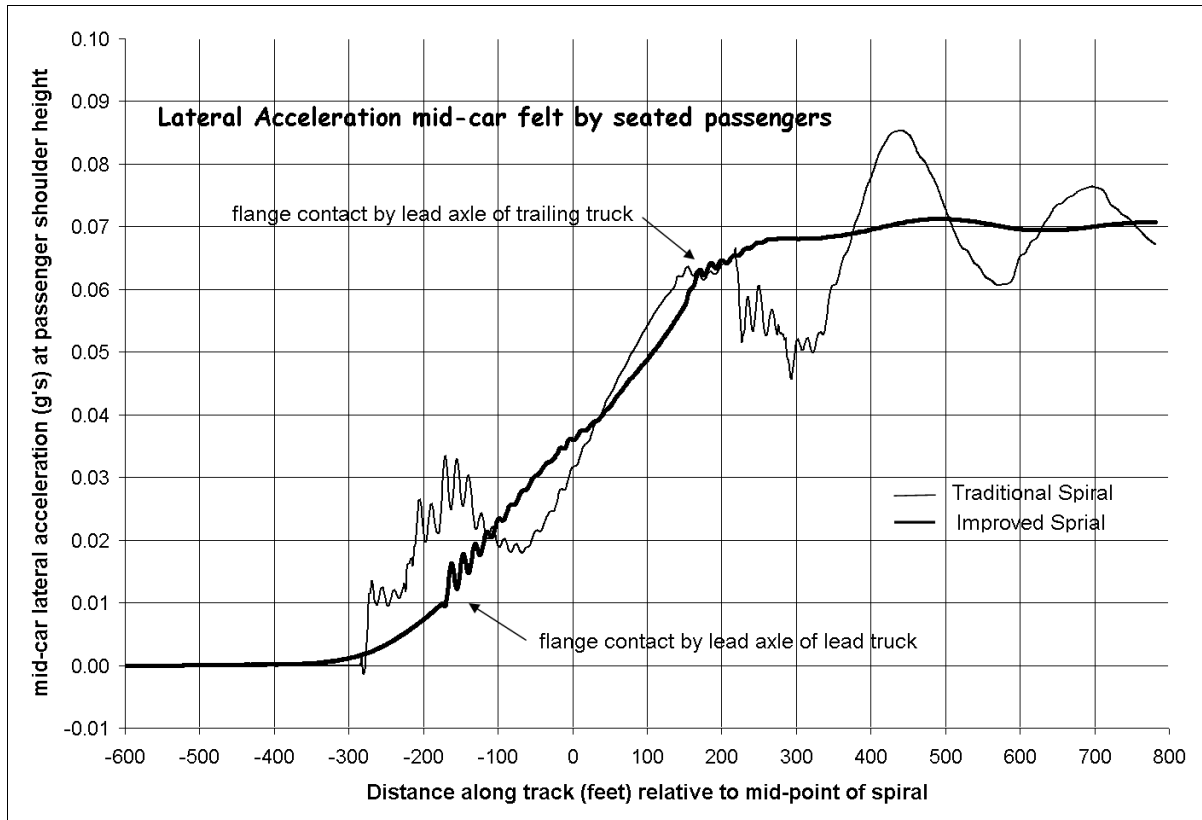


FIGURE 7 Predicted car body lateral accelerations 2.13 m (7 ft) above track at mid-car for an ACELA coach traveling at 114 mph over a traditional linear spiral and over an order {2,1} spiral with 2.44 m (8 ft) roll axis height. Both spirals connect tangent track to same 1.0 deg curve elevated for balance at 90 mph. Active tilt of the ACELA is turned off.

The predicted sums of the lateral forces applied by all four wheels of the lead and trailing trucks are shown in Figures (8) and (9).

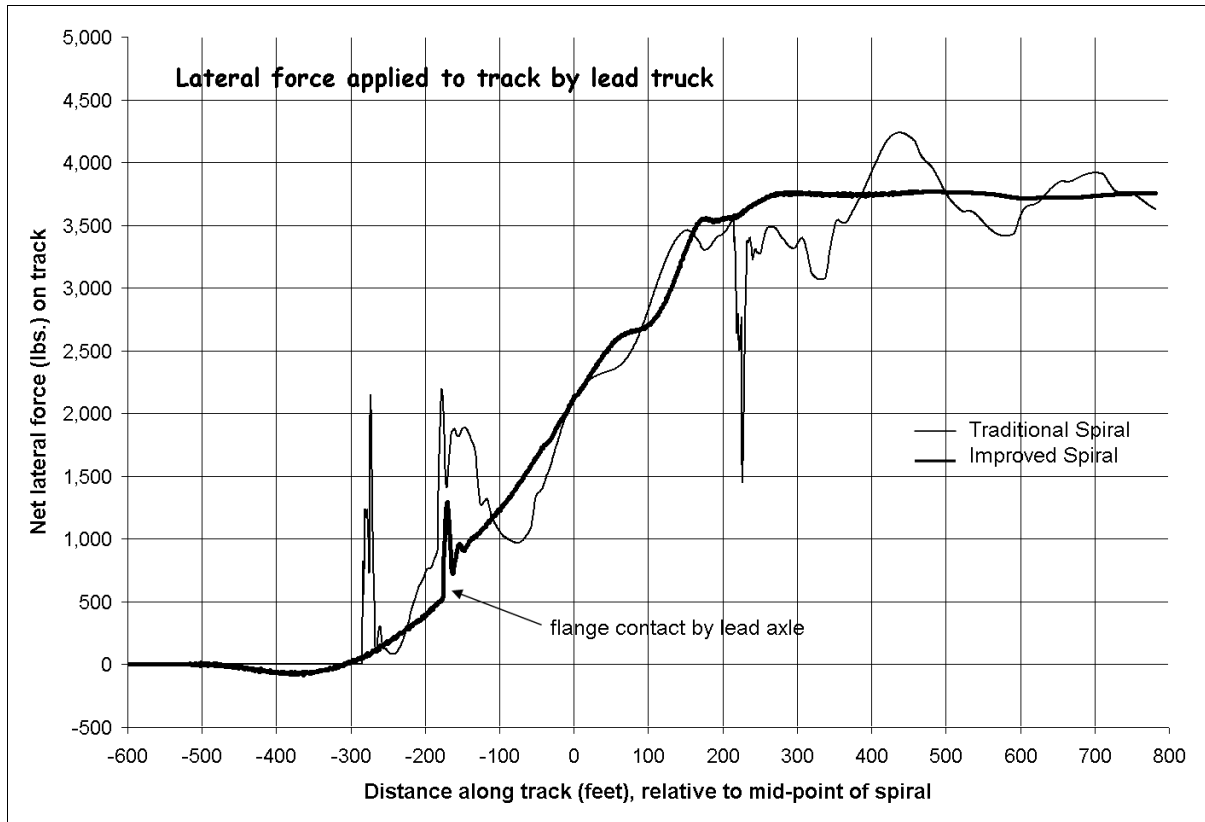


FIGURE 8 Predicted net lateral forces applied to track by lead truck of an ACELA coach traveling at 114 mph over a traditional linear spiral and over a corresponding order {2,1} spiral with 2.44 m (8 ft) roll axis height.

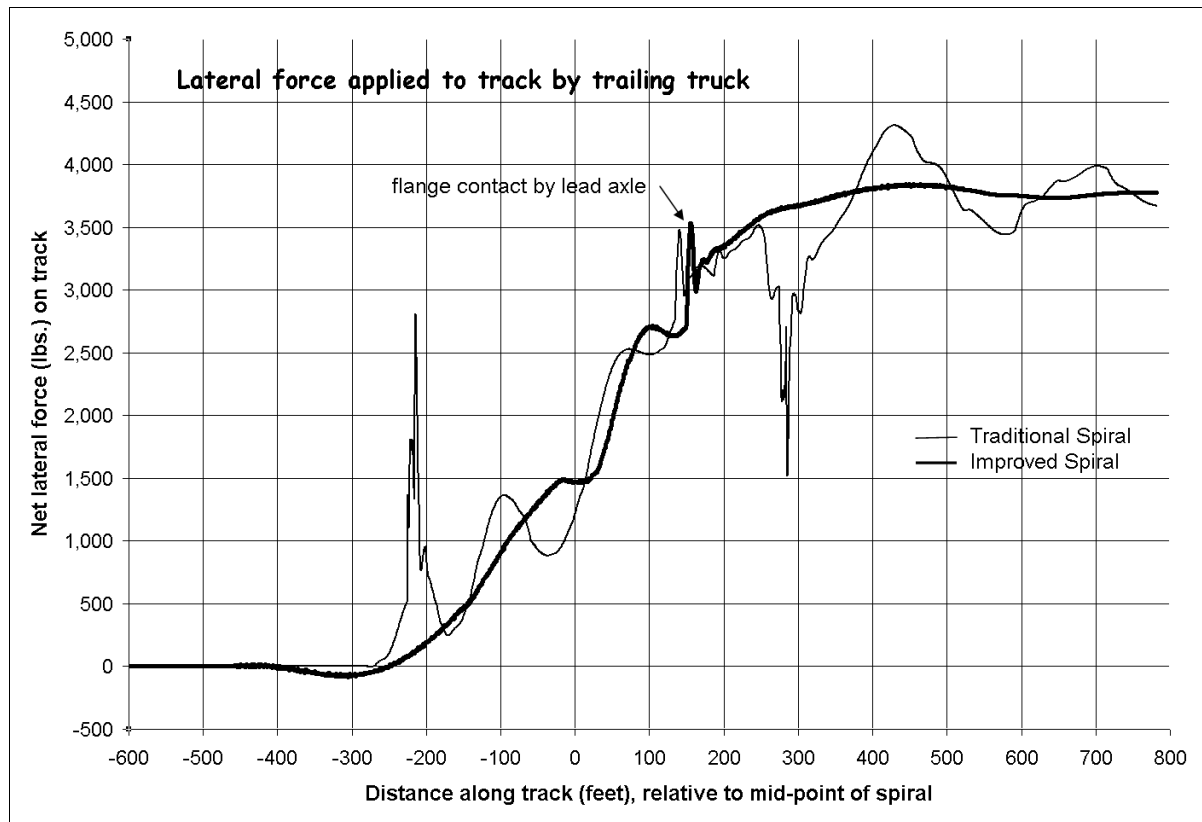


FIGURE 9 Same as figure 8 but for trailing truck.

Figures 7 through 9 show that the fluctuations of lateral car body acceleration and track forces present in the predicted dynamic behavior with the improved spiral are much weaker than their counterparts with the traditional linear spiral. Figure (7) shows that in the simulation for the improved spiral most of the remaining fluctuation in car-body acceleration occurs in two episodes of oscillation. We would like to see what causes these oscillations.

In this context, higher frequency oscillations are often associated with behavior of the trucks. Attention was therefore directed to the simulation results for the paths of the wheelsets within the track gauge and for the yaw angles of the trucks. As a truck enters the spiral and encounters increasing track curvature, its wheelsets move toward the outside or “high” rail. As a wheelset moves toward the high rail the conicity of the wheel treads generates torque about the yaw axis that tries to steer the wheelset away from the high rail. In the simulation reported here the truck does not respond to these torques quickly enough to prevent the high wheel flange of the leading wheelset of each truck from hitting the gauge corner of the high rail. Thus, flange impact occurs, and that impact appears to cause the observed peaks in lateral force applied to the track and to cause the observed car-body oscillation.

Figure 10 shows the lateral displacements of the wheelsets within the track gauge for simulated travel over the order {2,1} spiral.

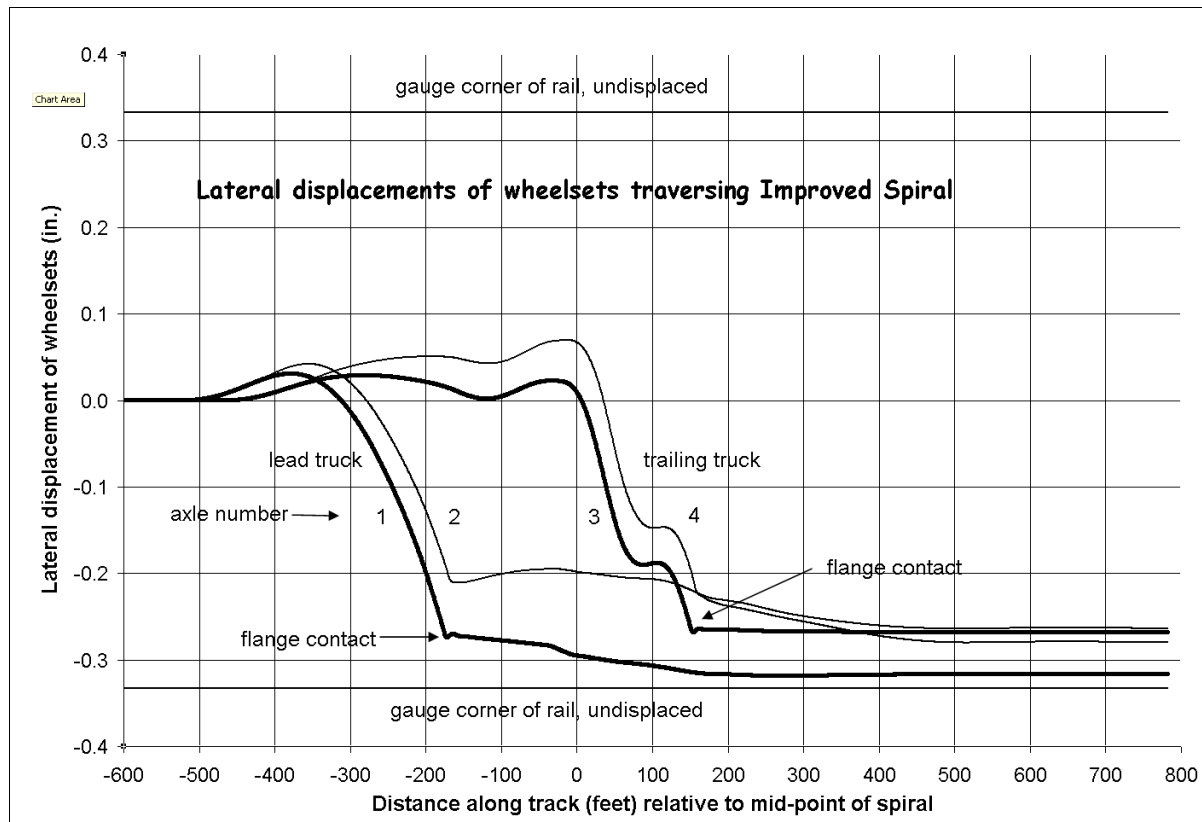


FIGURE 10 Predicted wheelset lateral displacements within track gauge for the four axels of an ACELA coach traveling at 114 mph over an order {2,1} spiral with 2.44 m (8 ft) roll axis height.

The lead high wheel flange of the lead truck (labeled on Figure 10 as axle 1) appears to hit the high rail when the mid point of the spiral is about 175 feet ahead of the middle of the coach. A peak in the lateral force applied to the track by the lead truck can be seen in Figure 8 at the same location. That is the location of the onset of the first episode of car-body oscillation seen in Figure 7.

The lead high wheel flange of the trailing truck (labeled on Figure 10 as axle 3) appears to hit the high rail when the middle of the coach is about 160 ft beyond the mid point of the spiral. A peak in the lateral force applied to the track by the trailing truck can be seen in Figure 9 at that location. That is the location of the onset of the second episode of car-body oscillation seen in Figure 7.

Figure 11 shows the yaw angles of the two trucks. The abrupt rise in the lead truck yaw angle at about - 175 ft. and the abrupt rise in the trailing truck yaw angle at about 160 ft. correspond to the flange hits as just described.

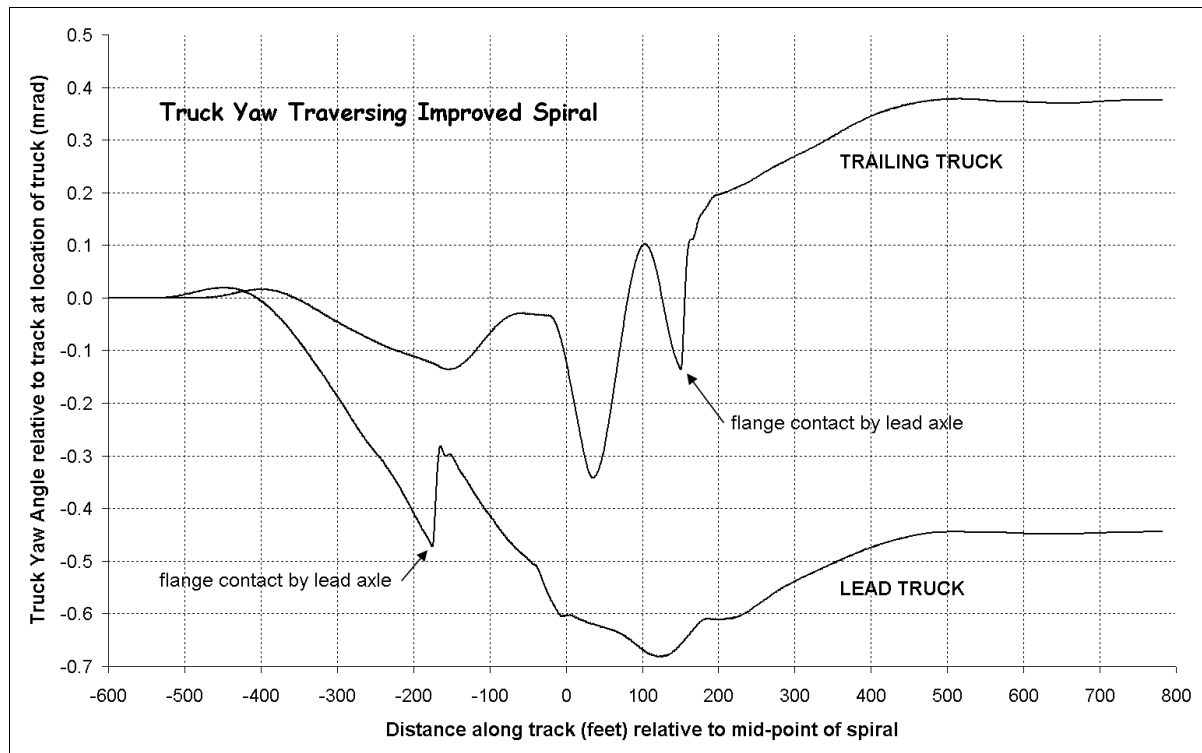


FIGURE 11 Predicted yaw angle of each truck of an ACELA coach traveling at 114 mph over an order {2,1} spiral with 2.44 m (8 ft) roll axis height. (Yaw angle is relative to the direction of the track at the location of the truck.)

There remains the question of how the flange impacts illustrated in figures (9) through (11) might be eliminated. The simulated results shown in figures (8) and (9) suggest that flange impact behavior is not significantly affected by change of spiral shape that does not include change of spiral offset. Increasing spiral offset and thereby making spirals much longer would have a beneficial effect in situations (such as new construction) where that may be practical. However, another interesting possibility is that when simulations like those reported here are done for a vehicle with trucks (such as so-called radial steering trucks) that respond rapidly to changing curvature, then the flange impacts shown above may be absent. If that level of truck steering capability is realized, then the motion of a vehicle so equipped over the improved spiral will be very smooth.

CONCLUSIONS

We have shown that changing spiral geometry from the traditional linear form to an improved form denoted here as order {2,1} with 2.44 m (8 ft) roll axis height brings about a substantial improvement in simulated dynamic performance. In the context of this paper, changing to the improved spiral shape reduces spiral induced fluctuation in lateral acceleration felt by passengers to about $1/6^{\text{th}}$ of the former value and reduces fluctuation in lateral force applied by vehicle trucks to the track structure to about $1/4^{\text{th}}$ of its former value. Almost all of the undesirable passenger acceleration and track force fluctuations that remain with the improved spiral shape appear to be due to slowness of steering response on the part of the vehicle trucks that were simulated. It should be kept in mind that this is a simulated result dependent on numerous assumptions and that actual behavior of the ACELA truck may

be different. It will be of interest to simulate trucks with more rapid steering response with the expectation that still better dynamic performance can be obtained. In order for the improvements predicted in this paper to be realized in practice, the surface and crosslevel of the track in question need to be of high quality.

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