American Railway Engineering and Maintenance-of-Way Association
Ballot

1. **Committee and Subcommittee: 8**
2. **Ballot Number: 08-21-01**
3. **Assignment:** Edits to Part 2 - Rho Max
4. **Ballot Item:**
5. **Rationale:**

Note to Publisher - REMOVE THIS LINE FROM 2.2.1 Notations

ρ_b = reinforcement ratio producing balanced strain conditions. See Article 2.32.1

LOAD FACTOR DESIGN

(APPLICABLE TO Section 2.30 THROUGH Section 2.39)

SECTION 2.30 STRENGTH REQUIREMENTS

2.30.1 REQUIRED STRENGTH (2005) R(2022)

Structures and structural members shall be designed to have design strengths at all sections at least equal to the required strengths calculated for the factored loads and forces in such combinations as stipulated in Article 2.2.4c, which represent various combinations of loads and forces to which a structure may be subjected. Each part of such structure shall be proportioned for the group loads that are applicable, and the maximum design required shall be used. Members shall also follow all other requirements of this Chapter to ensure adequate performance at service load levels.

2.30.2 DESIGN STRENGTH (~~2021~~2023) ~~R(2022)~~

a. For reinforced concrete members designed with reference to load factors and strengths, the design strength provided by a member, its connections to other members, and its cross sections, in terms of flexure, axial load, and shear, shall be taken as the nominal strength calculated in accordance with the requirements and assumptions of LOAD FACTOR DESIGN, multiplied by a strength reduction factor ϕ.

~~Strength reduction factor ϕ shall be taken as follows:~~

~~For flexure ϕ = 0.90~~
~~For shear and torsion ϕ = 0.85~~
~~For spirally reinforced compression members, with or without flexure ϕ = 0.75~~
~~For tied reinforced compression members with or without flexure ϕ = 0.70~~
~~**NOTE:** The value of ϕ may be increased linearly from the value for compression members to the value for flexure as the axial load strength P_n decreases from P_b to zero.~~
~~For bearing on concrete ϕ = 0.70~~

b. The provisions of this article are applicable to reinforced concrete sections with non-prestressed reinforcement elements having specified yield strengths up to 60,000 psi (420 MPa). Strength reduction factor ϕ shall be taken as follows:

For tension-controlled reinforced concrete sections	ϕ= 0.90
For shear and torsion in reinforced concrete sections	ϕ= 0.90
For compression-controlled sections with spirals or ties	ϕ= 0.75
For bearing on concrete	ϕ= 0.70
For compression in strut-and-tie models	ϕ= 0.70
For tension in strut-and-tie models	ϕ= 0.90

For sections in which the net tensile strain in extreme tension steel at normal resistance is between the compression-controlled strain limit, ε_{cl}, and tension-controlled strain limit, ε_{tl}, the value of ϕ associated with net tensile strain may be obtained by a linear interpolation as shown in Figure 8-2-4.

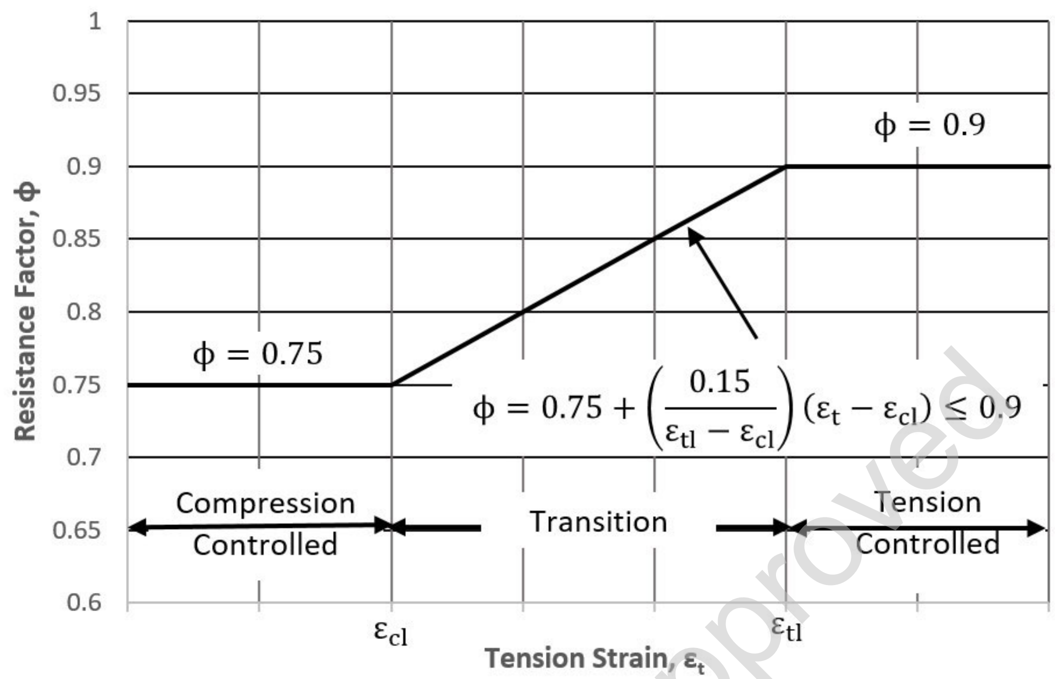

Figure 8-2-4 Variation of ϕ with Net Tensile Strain, ε_t, for Non-Prestressed Reinforcement

where:

ε_t = net tensile strain in the extreme tension steel at nominal resistance, in/in (mm/mm)
ε_{cl} = compression-controlled strain limit in the extreme tension steel, in/in (mm/mm)
ε_{tl} = tension-controlled strain limit in the extreme tension steel, in/in (mm/mm)

Where combinations of different grades of reinforcement are used in design, the lowest resistance factor calculated for each grade of reinforcement shall be used.

NOTE: Development and splices of reinforcement specified in Section 2.13 through Section 2.22 do not require a ϕ factor.

SECTION 2.31 DESIGN ASSUMPTIONS

2.31.1 STRENGTH DESIGN (20052023) R(2022)

Strength design of members for flexure and axial loads shall be based on the assumptions given in this article, and on satisfaction of the applicable conditions of equilibrium and compatibility of strains.

a. Strain in the reinforcing steel and concrete shall be assumed directly proportional to the distance from the neutral axis, except for deep members which shall satisfy the requirements for disturbed regions.

b. If the concrete is unconfined, the mMaximum usable strain at the extreme concrete compression fiber shall be assumed equal to 0.003.

c. If the concrete is confined, a maximum usable strain exceeding 0.003 in the confined core may be utilized if verified. Calculation of the factored resistance shall consider that the concrete cover may be lost at strains compatible with those in the confined concrete core.

d. The relationship between concrete compressive stress distribution and concrete strain may be assumed to be a rectangle, trapezoid, parabola, or any other shape which results in prediction of strength in substantial agreement with the results of comprehensive tests. These requirements may be considered satisfied by an equivalent rectangular concrete stress distribution defined as follows: A concrete stress of 0.85 f'_c shall be assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a straight line located parallel to the neutral axis at a distance ($a = \beta_1 c$) from the fiber of maximum compressive strain. The distance from the fiber of maximum strain to the neutral axis, "c", is measured in a direction perpendicular to that axis. The factor β_1 shall be taken as 0.85 for concrete strength f'_c up to and including 4000 psi (28 MPa). For strengths above 4000 psi (28 MPa) β_1 shall be reduced continuously at a rate of 0.05 for each 1000 psi (7 MPa) of strength in excess of 4000 psi (28 MPa), but β_1 shall not be taken less than 0.65.

a.e. Tensile strength of concrete shall be neglected in flexural calculations of reinforced concrete.

f. Balanced strain conditions exist at a cross section when tension reinforcement reaches the strain corresponding to its specified yield strength f_y just as the concrete compression reaches its assumed ultimate strain of 0.003.

b.g. Stress in reinforcement below the specified yield strength f_y for the grade of steel used shall be taken as E_s times the steel strain. For strains greater than that corresponding to f_y, the stress in the reinforcement shall be considered independent of strain and equal to f_y.

Tensile strength of concrete shall be neglected in flexural calculations of reinforced concrete.

The relationship between concrete compressive stress distribution and concrete strain may be assumed to be a rectangle, trapezoid, parabola, or any other shape which results in prediction of strength in substantial agreement with the results of comprehensive tests.

c. Balanced strain conditions exist at a cross section where tension reinforcement reaches the strain corresponding to its specified yield strength f_y just as the concrete compression reaches its assumes ultimate strain of 0.003.

d. The requirements of Article 2.31.1ef may be considered satisfied by an equivalent rectangular concrete stress distribution defined as follows: A concrete stress of 0.85 f'_c shall be assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a straight line located parallel to the neutral axis at a distance ($a = \beta_1 c$) from the fiber of maximum compressive strain. The distance c from the fiber of maximum strain to the neutral axis is measured in a direction perpendicular to that axis. The factor β_1 shall be taken as 0.85 for concrete strength f'_c up to and including 4000 psi (28 MPa). For strengths above 4000 psi (28 MPa) β_1 shall be reduced continuously at a rate of 0.05 for each 1000 psi (7 MPa) of strength in excess of 4000 psi (28 MPa), but β_1 shall not be taken less than 0.65.

h. Sections are compression-controlled where the net tensile strain in the extreme tension steel is equal to or less than the compression-controlled strain limit, ε_{cl}, at the time the concrete in compression reaches its assumed strain limit of 0.003. The compression-controlled strain limit is the net tensile strain in the reinforcement at balanced strain conditions. For reinforcement with a specified minimum yield strength of f_y <= 60,000 psi (420 MPa), ε_{cl} is taken as f_y/E but not greater than 0.002.

i. Sections are tension-controlled where the net tensile strain in the extreme tension steel is equal to or greater than the tension-controlled strain limit ε_{tl}, just as the concrete in compression reaches its assumed strain limit of 0.003. Sections with net tensile strain in the extreme tension steel between the compression-controlled strain limit and the tension-controlled strain limit constitute a transition region between compression-controlled and tension-controlled. The tension-controlled strain limit, ε_{tl}, shall be taken as 0.005 for reinforcement with a specified minimum yield strength. See 2.30.2b for the Strength reduction factor ϕ.

j. The use of compression reinforcement in conjunction with additional tension reinforcement is permitted to increase the strength of flexural members.

e.

SECTION 2.32 FLEXURE

2.32.1 DETERMINATION OF NEUTRAL AXIS LOCATION (2023)

For rectangular sections with tension reinforcement and with or without compression reinforcement,

$$c = \frac{A_s f_s - A'_s f'_s}{0.85 f'_c \beta_1 b}$$

For T-shaped compression block sections with tension reinforcement and with or without compression reinforcement,

$$c = \frac{A_s f_s - A'_s f'_s - 0.85 f'_c (b - b_w) h_f}{0.85 f'_c \beta_1 b_w}$$

Where

A_s	=	area of nonprestressed tensile reinforcement, in^2 (mm^2)
A_s'	=	area of nonprestressed compression reinforcement, in^2 (mm^2)
b	=	width of the compression face of the member. For a t-shaped section, the flange width, inches (mm)
b_w	=	web width, inches(mm)
c	=	neutral axis depth measured from the extreme compression fiber, inches(mm)
f_c'	=	specified concrete compressive strength, psi (MPa)
f_s	=	stress in nonprestressed tension reinforcement, psi (MPa)
f_s'	=	stress in nonprestressed compression reinforcement, psi (MPa)
h_f	=	compression flange depth, in.(mm)
β_1	=	stress block factor, see 2.31.1d

2.32.2 DETERMINATION OF STRESS IN REINFORCING STEEL (2023)

In the approximate flexural strength equations of 2.32.3 and 2.32.4, f_s and f_s' may be replaced with f_y and f_y' subject to the following conditions:

For tension reinforcement, f_s may be replaced with f_y as long as the value for c/d_s does not exceed

$$\frac{c}{d_s} \leq \frac{0.003}{0.003 + \varepsilon_{cl}}$$

Where

c = distance from the extreme compression fiber to the neutral axis, inches(mm)

d_s = distance from the extreme compression fiber to the centroid of the tension nonprestressed reinforcement, inches(mm)

ε_{cl} = compression controlled strain limit, see 2.31.1h

If c/d_s exceeds this limit, use strain compatibility to determine the stress in the nonprestressed tension reinforcement.

For compression reinforcement, f_s' may be replaced with f_y' if $c \geq 3d_s'$ and $f_y \leq 60{,}000$ psi(420 MPa); otherwise strain compatibility shall be used to determine the stress in the nonprestressed compression reinforcement. Alternately, the compression reinforcement can be conservatively ignored.

Where

d_s' = distance from extreme compression fiber to centroid of compression reinforcement, inches(mm)

a. **MAXIMUM REINFORCEMENT OF FLEXURAL MEMBERS (1992) R(2022)**

b.

c. For flexural members, the reinforcement ρ provided shall not exceed 0.75 of that ratio ρ_b which would produce balanced strain conditions for the section under flexure.

d.

e. For flexural members with compression reinforcement, the portion of ρ_b balanced by compression reinforcement need not be reduced by the 0.75 factor.

f.

g. Balanced strain conditions exist at a cross section when the tension reinforcement reaches its specified yield strength f_y just as the concrete in compression reaches its assumed ultimate strain of 0.003.

2.32.22.32.3 RECTANGULAR SECTIONS WITH TENSION OR TENSION AND COMPRESSION REINFORCEMENT ONLY (20222023)

For tension-controlled rectangular sections with tension or tension and compression reinforcement, , when $\rho \leq 0.75 \rho_b$ the nominal flexural resistancedesign moment strength Φ M_n may be computed by:

$$\Phi M_n = \Phi\left[A_s f_y d\left(1 - \frac{0.6\rho f_y}{f'_c}\right)\right]$$ **EQ 2-28**

$$\Phi M_n = \Phi\left[A_s f_y\left(d - \frac{a}{2}\right)\right]$$ **EQ 2-29**

where:

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s f_s\left(d_s - \frac{a}{2}\right) - A'_s f'_s\left(d'_s - \frac{a}{2}\right)$$ **EQ 2-28**

$$M_u \leq \phi M_n$$

Where

a	=	$\beta_1 c$, inches(mm); depth of the equivalent stress block, See 2.32.1 for "c" and 2.31.1d for "β_1"
Φ	=	strength reduction factor, see 2.30.2

a.

b. The balanced reinforcement ratio ρ_b for rectangular sections with tension reinforcement only is given by:

$$\rho_b = \frac{0.85\,\beta_1 f'_c}{f_y}\left(\frac{87{,}000}{87{,}000 + f_y}\right)$$ **EQ 2-30**

$$\rho_b = \frac{0.85\,\beta_1 f'_c}{f_y}\left(\frac{600}{600 + f_y}\right)$$ **EQ 2-30M**

2.32.32.32.4 I- AND T-SECTIONS WITH TENSION OR TENSION AND COMPRESSION REINFORCEMENT ONLY (20052023) R(2022)

a. When the compression flange thickness is equal to or greater than the depth of the equivalent rectangular stress block "a" and the section is tension-controlled$\rho \leq 0.75\ \rho_b$, the nominal flexural resistancedesign moment strength Φ M_n may be computed by the equations given in Article 2.32.23.

b. When the compression flange thickness is less than "a" and the section is tension-controlled, the nominal flexural resistancedesign moment strength Φ M_n may be computed by:

$$M_n = A_s f_s \left(d_s - \frac{a}{2}\right) - A'_s f'_s \left(d'_s - \frac{a}{2}\right) + 0.85 f'_c (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2}\right) \qquad \textbf{EQ 2-29}$$

$$M_u \leq \phi M_n$$

Where

a = $\beta_1 c$, inches(mm); depth of the equivalent stress block, See 2.32.1 for "c" and 2.31.1d for "β_1"

Φ = strength reduction factor, see 2.30.2

$$\Phi M_n = \Phi\left[(A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right) + A_{sf} f_y (d - 0.5 h_f)\right] \qquad \textbf{EQ 2-31}$$

where:

$$A_{sf} = 0.85\, f'_c (b - b_w) \frac{h_f}{f_y}$$

c. The balanced reinforcement ration ρ_b for I- and T- sections with tension reinforcement only is given by:

$$\rho_b = \frac{b_w}{b}\left[\frac{0.85\, \beta_1 f'_c}{f_y}\left(\frac{87{,}000}{87{,}000 + f_y}\right) + \rho_f\right] \qquad \textbf{EQ 2-32}$$

$$\rho_b = \frac{b_w}{b}\left[\frac{0.85\, \beta_1 f'_c}{f_y}\left(\frac{600}{600 + f_y}\right) + \rho_f\right] \qquad \textbf{EQ 2-32M}$$

where:

$$\rho_f = \frac{A_{sf}}{b_w d}$$

~~d. When the compression flange thickness is greater than a, the design moment strength, ΦM_n, may be computed by using the equations in Article 2.32.2.~~

~~e.~~c. For T-girder and box-girder construction defined by Article 2.23.10 and Article 2.23.11, the width of the compression face, "b", shall be equal to the effective slab width.

~~2.32.4 RECTANGULAR SECTIONS WITH COMPRESSION REINFORCEMENT (2005) R(2022)~~

~~a. For tension controlled rectangular sections with compression reinforcementwhen $\rho \leq 0.75\ \rho_b$, the design moment strength ΦM_n may be computed by:~~

$$\Phi M_n = \Phi\left[(A_s - A'_s)f_y\left(d - \frac{a}{2}\right) + A'_s f_y (d - d')\right] \quad \text{EQ 2-33}$$

~~where:~~

$$a = \frac{(A_s - A'_s)\,f_y}{0.85\, f'_c\, b}$$

~~and the following condition shall be satisfied:~~

$$\frac{A_s - A'_s}{bd} \geq \frac{0.85\,\beta_1 f'_c\, d'}{f_y d}\left(\frac{87{,}000}{87{,}000 - f_y}\right) \quad \text{EQ 2-34}$$

$$\frac{A_s - A'_s}{bd} \geq \frac{0.85\,\beta_1 f'_c\, d'}{f_y d}\left(\frac{600}{600 - f_y}\right) \quad \text{EQ 2-34M}$$

~~b. When the value of $(A_s - A'_s)/bd$ is less than the value given by EQ 2-34, so that the stress in the compression reinforcement is less than the yield strength f_y or when effects of compression~~

reinforcement are neglected, the moment strength may be computed by the equations in Article 2.32.21, except when a general analysis is made based on stress and strain compatibility using the assumptions given in Section 2.31.

The balanced reinforcement ratio ρ_b for rectangular section with compression reinforcement is given by:

$$\rho_b = \frac{0.85\,\beta_1 f'_c}{f_y}\left(\frac{87{,}000}{87{,}000+f_y}\right) + \frac{\rho' f'_{sb}}{f_y} \qquad \textbf{EQ 2-35}$$

$$\rho_b = \frac{0.85\,\beta_1 f'_c}{f_y}\left(\frac{600}{600+f_y}\right) + \frac{\rho' f'_{sb}}{f_y} \qquad \textbf{EQ 2-35M}$$

where:

f'_{sb} is stress in compression reinforcement at balanced strain conditions

$$f'_{sb} = 87{,}000 - \frac{d'}{d}\left(87{,}000 + f_y\right) \le f_y$$

$$f'_{sb} = 600 - \frac{d'}{d}\left(600 + f_y\right) \le f_y \qquad \text{(metric)}$$

2.32.5 OTHER CROSS SECTIONS (19922023) R(2022)

For other cross sections, the nominal flexural resistancedesign moment strength Φ M_n, shall be computed by a general analysis based on stress and strain compatibility using the assumptions given in Section 2.31. The requirements of Article 2.32.1 shall also be satisfied.