American Railway Engineering and Maintenance of Way Association
Letter Ballot

1. **Committee and Subcommittee:** Committee 5 Track, Subcommittee 8 Criteria for Track Geometry Design

2. **Letter Ballot Number:** 05-22-03

3. **Assignment:**

4. **Ballot Item:** Update Section 3.6 Vertical Curves

5. **Rationale:** The current 2021 MRE includes recommended practices for the design of vertical curves for freight, passenger, and transit lines. There are also provisions for calculation of slow speed curves. The previous revisions to the manual with respect to slow speed curves did not expand on the slow speed curve language. This revision adds more vertical curve language, replaces the cumbersome formula for finding a point on a vertical curve with a newer, easier to understand formula and adds elevations on curve calculation example with step-by-step instructions.

SECTION 3.6 VERTICAL CURVES (2023)

a. Vertical curves as calculated in item (g) below should be used to connect changes in gradients.

b. Use of a vertical curve is optional for changes in gradient (g1-g2, where g1 is the entrance gradient and g2 is the exit gradient) with an absolute value of less than 0.2 percent (absolute value of D less than 0.002).

c. Should a vertical curve be required, the length of vertical curve is determined by the amount of change in gradient, vertical acceleration and the speed of the train.

d. The purpose of the vertical curve is to ease the change of the gradients in order to reduce coupler and diaphragm binding and eliminate the danger of breaking trains in two as a direct result of train action. In addition, the proper vertical curve will provide for passenger comfort on passenger trains. Vertical curves should be designed as long as physically and economically possible.

e. A vertical curve which is concave upwards shall be denoted as a sag. A vertical curve which is concave downwards shall be denoted as a summit.

f. The vertical curve may be either circular or parabolic in shape.:

g. The **minimum** length of the vertical curve for both sags and summits is determined by the following formula:

$$L = \frac{D \times V^2 \times K}{A}$$

EQ 15

Where: A = vertical acceleration in feet/sec/sec (ft/sec^2)
D = Absolute value of the difference in rates of grades expressed as a decimal
K = 2.15 conversion factor to give L in feet
L = Length of vertical curve in feet
V = Speed of the train in miles per hour

h. The recommended vertical acceleration (A) should be selected based on the type of operations and is the same for both sags and summits.

Freight Operations:

A = 0.10 feet/sec/sec

Passenger and Transit Operations:

A = 0.60 feet/sec/sec

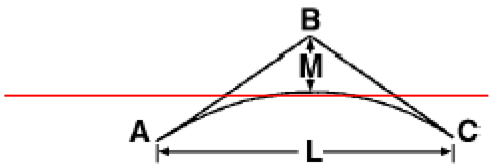
~~i. Curves constructed to this formula should not present any problems for the current generation of equipment.~~

j. The train speed which should be used in the above formula for establishing the length of vertical curve should be the maximum speed found on that particular subdivision or route. Special attention should be paid to locations where local conditions have dictated a speed restriction now in place, but where such a restriction might be removed at a later date.

k. It is not recommended to place turnouts and other special trackwork within the limits of a vertical curve.

~~l. Slow speed curves, such as hump crests, should, however, be designed with consideration for vertical clearance rather than using this formula.~~

~~m. One such form of vertical curve is developed as follows:~~



~~L = Length of vertical curve in 100-ft stations~~

~~M = Correction in elevation at B~~

~~M =~~

$$\frac{(\text{Elev B} \times 2) - (\text{Elev A} + \text{Elev C})}{4}$$

~~Whe~~ ~~ds –~~

~~M =~~

$$\frac{(\text{Elev A} + \text{Elev C}) - (\text{Elev B} \times 2)}{4}$$

~~n. The co~~ ~~ical curve is proportional to the square of its distance from A or C to B.~~

~~o. Corrections are – when the vertical curve is concave downwards and + when the vertical curve is concave upwards~~

Example Calculation for Freight Operations

Crest curve with 0.50% ascending grade meeting a 0.50% descending grade. Maximum design speed is 50 MPH.

A = 0.10 feet/sec/sec vertical acceleration (Freight)

D = Absolute value of ((+.005) - (-.005)) = 0.01

K = 2.15 conversion factor to give L in feet

V = 50 MPH design speed

$$L = \frac{D \times V^2 \times K}{A}$$ = Length of vertical curve in feet

$$L = \frac{(0.01) \times (50\text{MPH})^2 \times 2.15}{0.10 \ \text{feet/sec/sec}}$$ = 537.50 feet say 540 feet

Example Calculation for Passenger and Transit Operations

Sag curve with 0.50% descending grade meeting a 0.50% ascending grade. Maximum design speed is 75 MPH.

A = 0.60 feet/sec/sec vertical acceleration (Passenger and Transit)

D = Absolute value of ((-.005) - (+.005)) = 0.01

K = 2.15 conversion factor to give L in feet

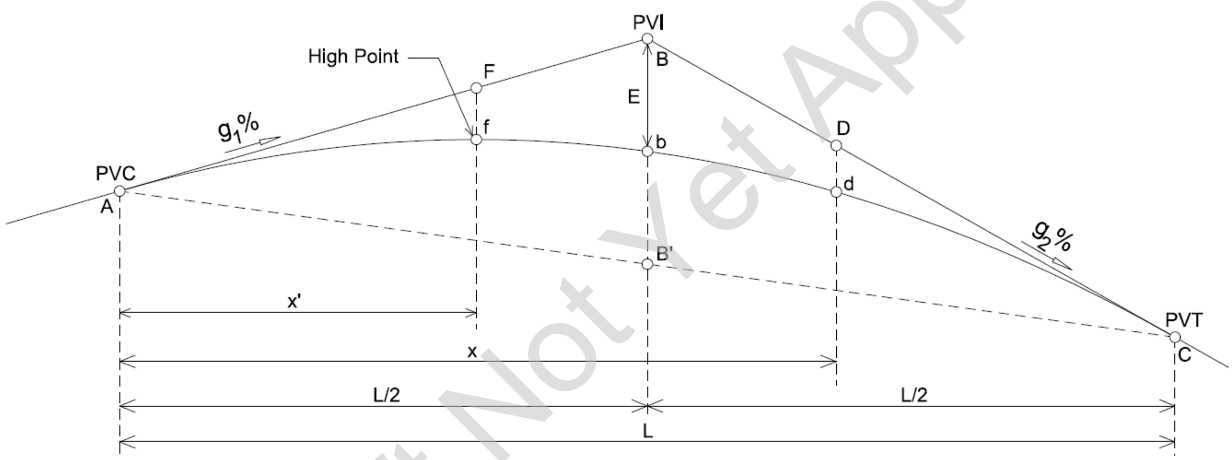
V = 75 MPH design speed

$$L = \frac{D \times V^2 \times K}{A}$$ = Length of vertical curve in feet

$$L = \frac{(0.01) \times (75MPH)^2 \times 2.15}{0.60 \text{ feet/sec/sec}}$$ = 201.56 feet say 205 feet

k. Curves constructed to EQ 15 as described in item g should not present any problems for the current generation of equipment. Slow speed curves such as hump crests and some industrial vertical curves should, however, be designed with consideration for the host railroad's minimum length of curve requirement, the utilized equipment's vertical clearance above rail, and/or the constructability of a short vertical curve rather than using EQ 15 as described in item g. Additionally, the vertical curve design shall not violate any regulatory requirements.

l. Top of rail clearance to overhead structures or below ground utilities or pipes in vertical curves is a frequent occurrence. Determining clearance to these items requires the top of rail elevation. The top of rail elevation at any point within a parabolic vertical curve is determined as follows:



Elevation of any point (d) on the curve can be defined by equation (equation number):

$$Z_d = Z_{PVC} + g_1 x + \frac{rx^2}{2}$$ EQ 16

$$r = \frac{g_2 - g_1}{L}$$ EQ 17

Where:

Z_d = elevation of any point on the curve, (feet)

Z_{PVC} = elevation of PVC, (feet)

g_1 and g_2 = grades prior and after PVI respectively, expressed algebraically, (decimals)

- g_1 and g_2 are positive (+) for ascending and negative (-) for descending grades

r = rate of change of grade, (per 1-feet)

x = horizontal distance from PVC to any point on the curve, (feet)

L = length of vertical curve, (feet)

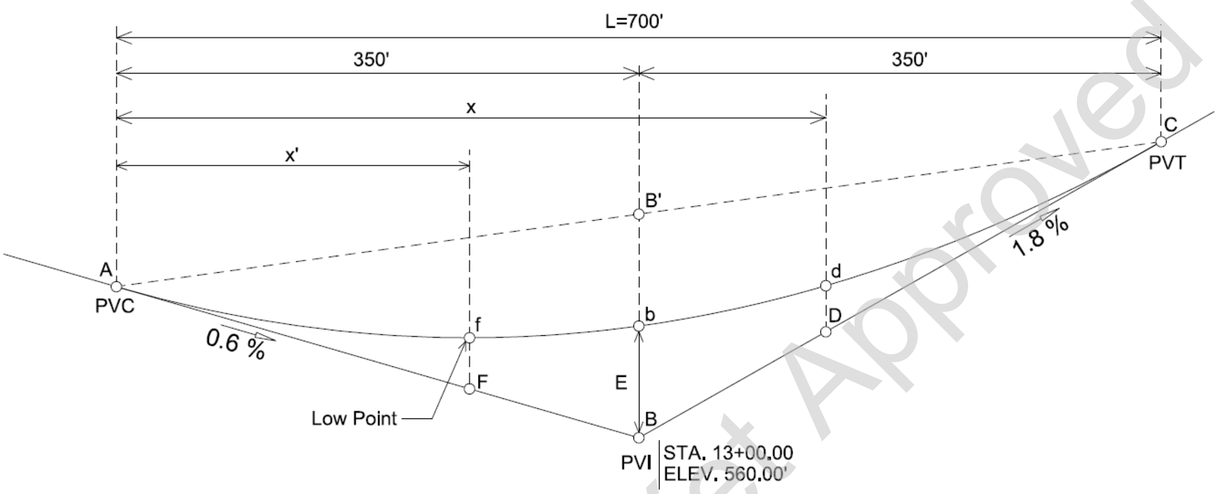
Vertical curves are classified as sag (concave upwards) curves where $g_2 > g_1$. Likewise, $g_2 < g_1$ represents crest (concave downwards) curves.

m. Distance of the high/low point (f) from PVC can be calculated using following formula:

$$x' = -\frac{g_1}{r} \qquad \text{EQ 18}$$

Where:
x' = distance of the high/low point from PVC, (feet)
g_1 = grade prior to PVI, expressed algebraically, (decimals)
r = rate of change of grade, (per 1-feet)
If $|g_1| = |g_2|$, then $x' = L/2$. That means low/high point on the vertical curve will be in the middle of curve.

Example Calculation for Top of Rail Elevation



For a sag curve, given:
$g_1 = -0.6\%$
$g_2 = +1.8\%$
$L = 700\ feet$
$PVI\ Station = 13 + 00.00$
$PVI\ Elevation = 560.00\ feet$

Determine the followings:
1. Station and elevation of PVC and PVT
2. Elevation of any point on the curve at 100-feet stations
3. Station and elevation of the low point

- **Station and elevation of PVC and PVT**

$PVC\ Sta, = PVI\ Sta, - L/2 = 1300 - (700/2) = 9 + 50{,}00$
$PVI\ Elev. = 560\ feet$
$350\ ft\ at\ 0.6\%\ grade = 2.1\ feet \rightarrow PVC\ Elev. = 560 + 2.1 = 562.1\ feet$

$PVT\ Sta, = PVI\ Sta, + L/2 = 1300 + (700/2) = 16 + 50{,}00$
$PVI\ Elev. = 560\ feet$
$350\ ft\ at\ 1.8\%\ grade = 6.3\ feet \rightarrow PVT\ Elev. = 560 + 6.3 = 566.3\ feet$

- **Elevation of any point on the curve at 100-ft stations** using equation 17:

$$r = \frac{g_2 - g_1}{L} = \frac{(0.018 - (-0.006))}{700} = \frac{3}{87500}$$

Use equation 16 to determine the elevation of any point on the curve:
For example, at Sta. 10+00.00:
$x = 1000 - 950 = 50\ feet$

$$Z_d = Z_{PVC} + g_1 x + \frac{rx^2}{2} = 562.1 + (-0.006)50 + \frac{3(50)^2}{87500 \times 2} = 561.84\ feet$$

Using same method, determine elevation for all 100-ft stations.

- **Station and elevation of the low point**

Using equation 22:

$$x' = -\frac{g_1}{r} = -\frac{-0.006}{\left(\frac{3}{87500}\right)} = 175\ feet$$

$$Low\ point\ Sta. = PVC\ Sta. + x' = 950 + 175 = 11 + 25.00$$

Using equation 16,

$$Z_{low\ point} = Z_{PVC} + g_1 x' + \frac{r(x')^2}{2} = 562.1 + (-0.006)175 + \frac{3(175)^2}{87500 \times 2} = 561.58\ feet$$

Summary of Results:

Point	Station	Curve Elevation (feet)
PVC	9 + 50.00	562.10
	10 + 00.00	561.84
	11 + 00.00	561.59
Low point	11 + 25.00	561.58
	12 + 00.00	561.67
PVI	13 + 00.00	562.10
	14 + 00.00	562.87
	15 + 00.00	563.99
	16 + 00.00	565.44
PVT	16 + 50.00	566.30