ABSTRACT

Load and Resistance Factor Design (LRFD) is a calibrated, reliability-based, limit-state methodology for structural design, as compared to the traditional, judgment-based, uncalibrated methods of Allowable Stress Design (ASD) or Load Factor Design (LFD). Factors for loads and resistances in the LRFD method are developed from current statistical information on loads and structural performance, by utilizing the theory of reliability. The first, calibrated, reliability-based, limit-state specifications for highway bridges in North America, was adopted in 1979 in Ontario, Canada. LRFD specifications in the USA had been available since 1986 for the design of steel structures and since 1994 for the design of highway bridges, but there are no such specifications for railroad bridges. This paper provides fundamental information on LRFD and examples on calculating the structural reliability of concrete and steel railroad bridges, and outlines a general strategy for developing and implementing LRFD specifications. Influence of live load bias and coefficient of variation on calculated reliability indices of the railroad bridges are shown.

Key Words: LRFD, Design Specifications, Railroad Bridges, Structures, Reliability.
INTRODUCTION

A general road map or action plan to develop and implement Load and Resistance Factor Design (LRFD) specifications with examples on calculating structural reliability of railroad bridges are presented and discussed in this paper. LRFD is a calibrated, reliability-based, limit-state method of structural design. This state-of-the art method provides a uniform level of structural reliability or safety amongst various bridge types, span lengths and load effects, as compared to non-uniform safety levels offered by the judgment-based traditional methods of Allowable Stress Design (ASD) and Load Factor Design (LFD).

A calibrated, limit-state code for highway bridge design was first published in 1979 in North America in Ontario, Canada (1). The first LRFD codes in the USA were published in 1986 for steel structures (2), 1994 for highway bridges (3) and 1996 for timber structures (4). A calibrated, strength-design method for concrete buildings was first published in 1971 (5). Subsequent editions to all of these codes have since been published. In the meantime railroad bridges continue to be designed with the traditional, safe methods of ASD and LFD (6) that have withstood the test of time.

A methodology for achieving LRFD railroad bridge specifications is presented. An established analytical process for developing and calibrating LRFD factors is summarized and illustrated through two examples of railroad bridges. LRFD development is presented in terms of technical objectives and organizational requirements.
LOAD AND RESISTANCE FACTOR DESIGN

Load and resistance factors in the LRFD method are developed from current statistical information or data on loads and structural performance and calibrated through the theory of reliability, to achieve a uniform level of safety against notional failure or a limit state being exceeded. A limit state in LRFD is a specified structural condition, beyond which a bridge or a component of a bridge ceases to satisfy its intended design function. There are four limits states, each with a corresponding set of load combinations. The limit states are:

- Service limit state that provides restrictions on stress, deformation and crack width
- Fatigue and fracture limit state that controls crack growth under repetitive load
- Strength limit state that provides strength and stability, locally and globally
- Extreme event limit state that ensures bridge survival from an earthquake or other rare-occurrence events

For each load combination in the four limit states, the following equation must be satisfied:

\[ \sum \eta_i \gamma_i Q_{ni} \leq \phi R_n \]  

(1)

where,

\[ R_n = \text{nominal resistance} \]
\[ \phi = \text{resistance factor} \]
\[ Q_{ni} = \text{nominal load effect} \]
\[ \gamma_i = \text{load factor} \]
\[ \eta_i = \text{load modifier} \]

The resistance factor \( \phi \) accounts for uncertainties in materials, fabrication and analysis. The load factor \( \gamma_i \) characterizes an inherent variability in design loads. The load modifier \( \eta_i \) accounts for ductility and redundancy in a bridge system and operational importance of the subject bridge, and is defined as

\[ \eta_i = \eta_D \eta_R \eta_I \geq 0.95 \quad \text{for maximum } \gamma_i \]  
\[ \eta_i = \frac{1}{\eta_D \eta_R \eta_I} \leq 1.0 \quad \text{for minimum } \gamma_i \]

where \( \eta_D, \eta_R, \eta_I \) are load modifiers for ductility, redundancy and operation importance.

For normal conditions, the load modifier is equal to one, and the limit state equation can be simplified and re-arranged as follows,

\[ \Sigma \gamma_i Q_{ni} \leq \phi R_n \]  
\[ \phi R_n - \Sigma \gamma_i Q_{ni} \geq 0 \]

**Reliability Concepts for Developing LRFD Factors**

Available statistical data on loads and resistances are characterized by the concepts of mean (or average), standard deviation, coefficient of variation (standard deviation divided
by mean), bias (measured value divided by nominal or designed value), as well as reliability index that represents the number of standard deviations below which there is a probability of notional failure occurring or a limit state being exceeded. All of these mathematical concepts are illustrated and simplified, to the extent possible, in this paper.

Normal statistical variation in loads or resistance can be characterized by the well-known “bell-shaped” curve as shown in Figure 1(a), where measured values of loads or resistance corresponding to frequency of occurrence (number of events) or probability of occurrence are drawn. A normal Probability Density Function (PDF) is represented in this figure and the mean or average of all measured values is shown.

A Cumulative Distribution Function (CDF) is shown in Figure 1(b), representing the same data in the previous figure, except that the cumulative probability of occurrence is developed. The maximum value of the CDF is one, which represents the total area under the PDF curve. The CDF or corresponding areas under the PDF curve represent the probability that the normal variable would be either less than or greater than a particular value. The vertical axis of this figure could be transformed to produce a straight-line behavior for normal distribution of data with the standard deviation as the slope, instead of the shown behavior with a changing slope, particularly near either end of the data.

A cumulative distribution function is plotted in Figure 2 using a transformed vertical axis that represents a standard normal variable ($z$) or normal inverse of probability of occurrence; the slope of the shown line is the inverse of the standard deviation $\sigma$ and the mean value of either load $Q_m$ or resistance $R_m$ corresponds to the zero value of the standard
normal variable. A “best-fit” line for data points is usually plotted. But non-linear behavior of data is possible and that would be an indication of lognormal distribution or other distributions, such as gamma and extreme types I, II and III.

Figure 3(a) shows separate PDF curves for loads and resistance, with mean and nominal values indicated on each curve. A nominal design load or a combination of such loads $Q_n$ is selected to be greater than the mean value of the load $Q_m$, whereas the nominal resistance $R_n$ is specified to be smaller than the mean value of the resistance $R_m$. A statistical variation of combined loads and resistance $(R - Q)$ is shown in Figure 3(b). Equation 3b is represented by this figure.

The indicated small area under the curve, where resistance is less than loads, represents a probability of notional failure occurring or a specified limit state being exceeded. The horizontal distance between the mean value and the boundary of notional failure is equal to the reliability index $\beta$ multiplied by the standard deviation $\sigma$. In other words, the reliability index of a particular design represents the number of standard deviations that the mean of the variable $(R - Q)$ is safely away from the design limit state. Mathematical expressions of reliability are provided in Appendix A.

**Comparing LRFD to Traditional Design Methods**

LRFD is significantly different from both LFD and ASD, even-though factored (increased) loads are compared with modified (reduced) strengths or resistances as in LFD and service loads are limited by some requirements that are similar to ASD. The essential difference is
that load and resistance factors in the LRFD method are based on extensive statistical data on loads and resistances and are calibrated through the same target reliability index, to achieve a uniform level of safety for various bridge types, span lengths and load effects.

With Allowable Stress Design (ASD), also referred to as Service Load Design (SLD) or Working Load Design (WLD), all loads on a structure are treated equally in terms of statistical variability and calculated stress effects are compared to allowable stresses, based on either ultimate stresses or yield stresses that are reduced by safety factors. Service stresses from load combinations considered less likely to occur, such as those with wind load, are compared with increased allowable stresses and a relatively smaller margin of safety is accepted for such load combinations. Past experience and engineering judgment had been the basis for determining the safety factors.

With Load Factor Design (LFD), also referred to as Strength Design or Ultimate Strength Design, all loads are increased by different factors (each larger than one) and calculated load effects are compared to slightly reduced ultimate strengths or capacities. Load variability, particularly live load as compared to dead load, is considered in this method. However, load and strength factors were not calibrated to achieve a uniform level of safety for all members of a bridge structure and amongst various types of bridges and span lengths. There is no rational guidance on adjusting the factors of this method to accommodate changed uncertainties in loads and strengths, as more research data becomes available.
EXAMPLES ON RELIABILITY INDICES FOR RAILROAD BRIDGES

Concrete Railroad Bridge

Consider a pre-stressed, concrete bridge that carries a single railroad track. The bridge structure has a span length of 29 ft (13.70 m) and consists of two adjacent, double-cell box-shaped beams. Beam width is 7 ft (3.31 m), beam depth is 2.5 ft (1.18 m) and ballast depth is 1.25 ft (0.59 m) maximum.

Table 1a shows calculated moments due to dead loads and live-plus-impact load, as well as assumed bias factors and coefficients of variation, with calculated means and standard deviations for each load. Of particular interest in this discussion are the bias factors and coefficients of variation for the various loads and the nominal resistance.

Assume for pre-cast (factory-made) beam weight that bias factor is 1.03 and the coefficient of variation is 0.08, based on data used in the calibration of LRFD for highway bridges (7). For the ballast weight, assume a bias factor of 1.05 and a coefficient of variation of 0.10, based on the same reference for cast-in-place members, even-though higher values could be justified due to the variability of this dead load. For miscellaneous weights (of track rails and others), assume similar statistical variation as for the ballast weight. For live-plus-impact load, assume a bias factor of 1.50 and a coefficient of variation of 0.15; data on statistical variation of the Cooper E 80 (EM 360) load and the impact load is not available.
For nominal resistance, assume a bias factor of 1.05 and a coefficient of variation of 0.075, based on data for pre-stressed concrete used in the calibration of LRFD for highway bridges (7). Nominal resistance is calculated from the following equation, by dividing the LFD load combination for dead load and live-plus-impact load by a resistance factor of 0.95, based on the AREMA Manual (6),

\[ R_n = \frac{1.4[D + (5/3)(L + I)]}{\phi} \]  \hspace{1cm} (4)

From Equation 18 in Appendix A, a reliability index for moment is calculated assuming that the parameter \( k \) is equal to two. Table 1b shows a calculated reliability index of 3.75. This value of the reliability index corresponds to a probability of notional failure of 1 in 10,000. If the beams are designed with higher nominal resistance than what is required by the loads, as is usually the case, the reliability index would increase and the probability of failure would decrease.

A target reliability index of 3.5 was used in the calibration of LRFD for highway bridges (7). The calculated reliability index of 3.75 in this example is highly dependent on the assumed bias factor and coefficient of variation of the train-plus-impact load. No significance should be given to the closeness of the two values.

Sensitivity of the reliability index \( \beta \) of this concrete railroad bridge to live (plus impact) load bias \( \lambda \) is shown in Figure 4(a). The reliability index decreases from a value of 6.47 for \( \lambda \) equals to one to a value of 1.81 for \( \lambda \) equals to two, with corresponding probabilities of
notional failure of 4 in $10^{11}$ and 3.5 in 100, respectively. The coefficient of variation $V$ of the live load is assumed to be equal to 0.15. The live load bias of two represents an unlikely possibility that the measured live-plus-impact load is twice the design train-plus-impact load. Figure 4(b) shows the influence of live load coefficient of variation $V$ on the reliability index, which decreases from a value of 6.12 for $V$ equals to one to a value of 2.31 for $V$ equals to 0.3, with corresponding probabilities of notional failure of 5 in $10^{10}$ and 1 in 100, respectively. A live load bias of 1.5 assumed. For the shown ranges of live load bias and coefficient of variation, the probabilities of notional failure are low. The design parameter $k$ within its typical range of values (1.5 to 2.5) and beyond has no significant influence on the reliability index, as shown in Figure 4(c).

**Steel Railroad Bridge**

Consider a steel bridge that supports a single railroad track. The bridge structure has a span length of 29 ft (13.70m), two I-shaped beams and an open deck. Beam spacing is 13.25 ft (4.04m) and beam web depth is 3.5 ft (1.65m). Table 2a shows calculated moments due to dead loads and live-plus-impact load, as well as assumed bias factors and coefficients of variation, with calculated means and standard deviations for each load.

Nominal resistance is calculated from the following equation, by dividing the ASD load combination for dead load and live-plus-impact load by a safety factor of 0.55, based on the AREMA Manual (6),

$$R_n = \frac{D + (L + I)}{SF}$$

(5)
Based on Equation 18 in Appendix A, a reliability index for moment is calculated assuming that the parameter $k$ is equal to two. Table 2b shows a calculated reliability index of 1.93. This value of the reliability index corresponds to a probability of notional failure of 2.7 in 100. Typically the design nominal resistance is higher than what is required by the loads, and therefore, the reliability index would be larger and the probability of failure would smaller. As stated for the concrete bridge example, the calculated reliability index is highly dependent on the assumed bias factor and coefficient of variation of the train-plus-impact load.

Influence of live (plus impact) load bias $\lambda$ on the reliability index $\beta$ of this steel railroad bridge is shown in Figure 5(a). The reliability index decreases from a value of 4.52 for $\lambda$ equals to one to a value of 0.16 for $\lambda$ equals to two, with corresponding probabilities of notional failure of 3 in $10^6$ and 4.4 in 10, respectively. The coefficient of variation $V$ of the live load is assumed to be equal to 0.15. The live load bias of two represents an unlikely scenario that the measured live-plus-impact load is twice the Cooper E 80 (EM 360) design load (including impact) and therefore the calculated low reliability index is not realistic.

Figure 5(b) shows the influence of live load coefficient of variation $V$ on the reliability index, which decreases from a value of 3.20 for $V$ equals to zero to a value of 1.14 for $V$ equals to 0.3, with corresponding probabilities of notional failure of 6.9 in 10,000 and 1.3 in 10, respectively. A live load bias of 1.5 is assumed. As shown in Figure 5(c), the design parameter $k$ within its typical range of values (1.5 to 2.5) and beyond has no significant influence on the reliability index.
WHY LRFD FOR RAILROAD BRIDGES?

LRFD is considered an appropriate design methodology based on its technical advantages over the traditional methods of ASD and LFD and for other reasons as listed below.

- LRFD is a calibrated, reliability-based, limit-state method of structural design, that offers a uniform level of safety for a variety of bridge types and span lengths and amongst various load effects.

- New construction technologies, materials and practices are increasingly influenced by LRFD specifications (10).

- The latest structural codes and highway bridge specifications for steel, concrete and timber (2-5) are based primarily on limit-state design or LRFD. While information taken from these sources is certainly reliable, the effect on traditional railroad bridge design practice in terms of relative reliability amongst bridge types, span lengths and other parameters is not known.

- Younger generations of structural engineers are being educated primarily in the LRFD method, with little emphasis on ASD and LFD. As prospective railroad bridge engineers, they may have to learn the traditional methods while on the job.

- Site-specific live load data and future changes in live loads could be incorporated into LRFD, using current statistical data and the concept of a target reliability index to provide uniform structural reliability.
ELEMENTS OF A ROAD MAP TO LRFD

Some specific steps for developing Load and Resistance Factor Design for railroad bridges are outlined below.

- Investigate the need for and feasibility of LRFD for railroad bridge design.
- Review methodologies of the various LRFD codes presently in use.
- Develop reliability-based load and resistance factors.
- Investigate the need for a new live load model.
- Develop separate factors for load rating or modify the developed design factors.
- Calibrate against existing and simulated railroad bridge designs.

Significant research would be needed for the development of new reliability-based factors and, if necessary, a new live load model for railroad bridge design. It should be noted that the relatively new AASHTO Manual for Condition Evaluation and Load and Resistance Factor Rating (LRFR) of Highway Bridges (9) has retained the previous Allowable Stress (AS) and Load Factor (LF) rating methods, as alternatives for load rating existing bridges. It is considered appropriate that existing bridges are rated with a method that is consistent with the original design.

LRFD flexural resistance requirements for steel bridges might be the most significant change from the current allowable stress format.
LRFD for Railroad Bridges

A pilot study, a comprehensive study and trial designs are required to develop LRFD specifications. A pilot study would serve as a preliminary phase for adjusting or fine-tuning the objectives of a comprehensive study.

An essential objective of a comprehensive study would be the development of calibrated factors for loads and resistances, which could be achieved through a calibration process similar to what was used for the development of the first AASHTO LRFD Specifications (3, 7). A calibration process would require the following considerations:

1. Select existing railroad bridges: An extensive database on actual railroad bridges would need to be collected. The selected bridges would need to represent various regions of North America and various bridge types, spans and materials within each region, as well as current and future trends of railroad bridge design. Load effects for the selected bridges would then be calculated and tabulated.

2. Develop simulated railroad bridges: A supplemental database on simulated railroad bridges might be necessary to evaluate a full range of bridge parameters. Such bridges would be designed based on the current AREMA Manual; only basic design calculations and tabulation of load effects would be required.

3. Collect statistical database for loads and resistances: It is considered that much of the data gathered during the AASHTO LRFD project might be useful for development of
LRFD for railroad bridges, except for live and impact loads. An extensive effort would be required to compile data on train and impact loads.

4. Develop new live load model: Depending on actual train load data, a new live load model might be necessary to represent the heaviest trains; though it is possible that the current Cooper E 80 (EM 360) live load could prove to be a realistic depiction of those trains.

5. Select a target reliability index: Reliability indices would need to be calculated for each of the selected existing bridges and the developed simulated bridges, based on the current AREMA Manual. Considering the performance of the evaluated bridges in terms of reliability, a target reliability index is selected to provide consistent and uniform safety margin for all bridge types, span lengths, load effects and other parameters.

6. Calculate load and resistance factors: Load factors are calculated so that factored loads would have a known, acceptable probability of being exceeded. The live load model would need to be used and resistance factors would need to be determined so that the reliability index would be equal approximately to the selected target value.

**CONCLUSIONS**

The requirements for the state-of-the art method of Load and Resistance Factor Design are investigated for the design of railroad bridges. LRFD is based on a limit-state philosophy, with calibrated, reliability-based factors for loads and resistances. Technical advantages of
LRFD over the traditional methods of ASD and LFD are discussed in this paper. A road map or a plan for development and implementation of LRFD is presented and examples on calculating reliability indices for railroad bridges are shown.

REFERENCES


(5) ACI, Building Code Requirements for Structural Concrete and Commentary, ACI Committee 318, American Concrete Institute, 1977-2002.


In limit-state design or LRFD, notional failure of a structural component is assumed not to occur if,

\[ g = R - Q \geq 0 \]  \hspace{1cm} (6)

where \( g \) is a random variable, \( Q \) is the load and \( R \) is the resistance. Equation 6 is similar to the simplified limit state Equation 3b.

If both \( R \) and \( Q \) are normal random variables, the standard deviation of the variable \( g \) and the reliability index \( \beta \) can be calculated as follows,

\[ \sigma_{R-Q} = \sqrt{\sigma_R^2 + \sigma_Q^2} \]  \hspace{1cm} (7)

\[ \beta = \frac{R_m - Q_m}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \]  \hspace{1cm} (8)

\[ \sigma_R = R_m V_R \]  \hspace{1cm} (9)

\[ \sigma_Q = Q_m V_Q \]  \hspace{1cm} (10)

where,

\( \sigma_R \) = standard deviation of resistance

\( \sigma_Q \) = standard deviation of load

\( V_R \) = coefficient of variation of resistance
\[ V_Q = \text{coefficient of variation of load} \]

\[ R_m = \text{mean value of resistance} \]

\[ Q_m = \text{mean value of load} \]

Mean resistance can be expressed in terms of bias factor \( \lambda_R \) and nominal resistance \( R_n \).

\[ R_m = \lambda_R R_n \quad (11) \]

Nominal resistance can be thought of in design terms as factored loads \( \Sigma \gamma_i q_i \) divided by a resistance factor \( \phi \),

\[ R_n = \frac{\Sigma \gamma_i q_i}{\phi} \quad (12) \]

Combine these Equations 11 and 12,

\[ R_m = \frac{\lambda_R \Sigma \gamma_i q_i}{\phi} \quad (13) \]

Re-arrange Equation 8 to solve for \( R_m \) and substitute from Equation 13,

\[ R_m = Q_m + \beta \sqrt{\sigma_R^2 + \sigma_Q^2} = \frac{\lambda_R \Sigma \gamma_i q_i}{\phi} \quad (14) \]

Solve for the resistance factor \( \phi \),

\[ \phi = \frac{\lambda_R \Sigma \gamma_i q_i}{Q_m + \beta \sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (15) \]
If $Q$ is a normal random variable and the resistance $R$ is a lognormal random variable, the reliability index $\beta$ is determined by combining Equation 8 with the following expressions for the mean and standard deviation of the resistance,

$$R_m = R_n \lambda_R (1 - k V_R [1 - Ln(1 - k V_R)])$$  \hspace{1cm} (16)

$$\sigma_R = R_n V_R \lambda_R (1 - k V_R)$$  \hspace{1cm} (17)

$$\beta = \frac{R_n \lambda_R (1 - k V_R [1 - Ln(1 - k V_R)]) - Q_m}{\sqrt{[R_n V_R \lambda_R (1 - k V_R)]^2 + \sigma_Q^2}}$$  \hspace{1cm} (18)

where $k$ is a parameter that depends on the location of a design point and varies typically from 1.5 to 2.5 and $Ln$ is a natural logarithmic function. The parameter $k$ represents the distance, in units of standard deviation, between a design point and the mean value.

Equations 9 through 15 are applicable to the case of normal $Q$ and lognormal $R$. Equation 18 is consistent with the reliability analysis used in the calibration of LRFD for highway bridges (7).
LIST OF TABLE TITLES

Table 1: (a) Statistical Variation of Calculated Load Effects on a Concrete Railroad Bridge
(b) Reliability Index Results

Table 2: (a) Statistical Variation of Calculated Load Effects on a Steel Railroad Bridge
(b) Reliability Index Results

LIST OF FIGURE CAPTIONS

Figure 1: (a) Normal Probability Density Function
(b) Cumulative Distribution Function

Figure 2: Cumulative Distribution Function Using Standard Normal Variable

Figure 3: (a) Probability Density Functions for Separated Load and Resistance
(b) Probability Density Function for Combined Load and Resistance

Figure 4: Sensitivity of Concrete Bridge Reliability Index to
(a) Live Load Bias,
(b) Live Load Coefficient of Variation and
(c) Design Parameter $k$

Figure 5: Sensitivity of Steel Bridge Reliability Index to
(a) Live Load Bias,
(b) Live Load Coefficient of Variation and
(c) Design Parameter $k$
<table>
<thead>
<tr>
<th>Load</th>
<th>$M$</th>
<th>$\lambda_Q$</th>
<th>$V_Q$</th>
<th>$Q_m$</th>
<th>$\sigma_Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>kip-ft (kN-m)</td>
<td></td>
<td>kip-ft (kN-m)</td>
<td>kip-ft (kN-m)</td>
<td></td>
</tr>
<tr>
<td>$D1$</td>
<td>159 (216)</td>
<td>1.03</td>
<td>0.08</td>
<td>164 (222)</td>
<td>13 (18)</td>
</tr>
<tr>
<td>$D2$</td>
<td>111 (150)</td>
<td>1.05</td>
<td>0.10</td>
<td>116 (158)</td>
<td>12 (16)</td>
</tr>
<tr>
<td>$D3$</td>
<td>35 (48)</td>
<td>1.05</td>
<td>0.10</td>
<td>37 (50)</td>
<td>3.7 (5.0)</td>
</tr>
<tr>
<td>$L+I$</td>
<td>1100 (1492)</td>
<td>1.50</td>
<td>0.15</td>
<td>1650 (2237)</td>
<td>248 (336)</td>
</tr>
<tr>
<td>$\Sigma D+L+I$</td>
<td>1405 (1905)</td>
<td></td>
<td></td>
<td>1967 (2667)</td>
<td>276 (374)</td>
</tr>
<tr>
<td>$\left[1.4\Sigma D+\frac{5}{3}(L+I)\right]\phi$</td>
<td>3152 (4273)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a)

<table>
<thead>
<tr>
<th>$R_n$</th>
<th>3152 (4273)</th>
<th>kip-ft (kN-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_g$</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$V_R$</td>
<td>0.075</td>
<td></td>
</tr>
<tr>
<td>$Q_m$</td>
<td>1967 (2667)</td>
<td>kip-ft (kN-m)</td>
</tr>
<tr>
<td>$\sigma_Q$</td>
<td>276 (374)</td>
<td>kip-ft (kN-m)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>3.75</td>
<td></td>
</tr>
</tbody>
</table>

(b)

Table 1: (a) Statistical Variation of Calculated Load Effects on a Concrete Railroad Bridge (b) Reliability Index Results
<table>
<thead>
<tr>
<th>Load</th>
<th>$M$</th>
<th>$\lambda_Q$</th>
<th>$V_Q$</th>
<th>$Q_m$</th>
<th>$\sigma_Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>(kip-ft)</td>
<td></td>
<td>(kip-ft)</td>
<td>(kip-ft)</td>
<td></td>
</tr>
<tr>
<td>$D1$</td>
<td>54 (75)</td>
<td>1.03</td>
<td>0.08</td>
<td>55 (75)</td>
<td>4.4 (6.0)</td>
</tr>
<tr>
<td>$D2$</td>
<td>0 (0)</td>
<td>1.05</td>
<td>0.10</td>
<td>0 (0)</td>
<td>0 (0)</td>
</tr>
<tr>
<td>$D3$</td>
<td>23 (31)</td>
<td>1.05</td>
<td>0.10</td>
<td>24 (33)</td>
<td>2.4 (3.3)</td>
</tr>
<tr>
<td>$L+I$</td>
<td>1132 (1534)</td>
<td>1.50</td>
<td>0.15</td>
<td>1697 (2301)</td>
<td>255 (345)</td>
</tr>
<tr>
<td>$\Sigma D+L+I$</td>
<td>1208 (1638)</td>
<td></td>
<td></td>
<td>1777 (2409)</td>
<td>261 (354)</td>
</tr>
<tr>
<td>$\Sigma [\Sigma D+(L+I)]/SF$</td>
<td>2196 (2978)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a)

<table>
<thead>
<tr>
<th>$R_n$</th>
<th>2196 (2978)</th>
<th>kip-ft (kN-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_g$</td>
<td>1.12</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$V_R$</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>$Q_m$</td>
<td>1777 (2409)</td>
<td>kip-ft (kN-m)</td>
</tr>
<tr>
<td>$\sigma_Q$</td>
<td>261 (354)</td>
<td>kip-ft (kN-m)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.93</td>
<td></td>
</tr>
</tbody>
</table>

(b)

Table 2: (a) Statistical Variation of Calculated Load Effects on a Steel Railroad Bridge (b) Reliability Index Results
Figure 1: (a) Normal Probability Density Function
(b) Cumulative Distribution Function
Figure 2: Cumulative Distribution Function Using Standard Normal Variable
Figure 3: (a) Probability Density Functions for Separated Loads and Resistance
(b) Probability Density Function for Combined Loads and Resistance
Figure 4: Sensitivity of Concrete Bridge Reliability Index to (a) Live Load Bias, (b) Live Load Coefficient of Variation and (c) Design Parameter $k$
Figure 5: Sensitivity of Steel Bridge Reliability Index to (a) Live Load Bias, (b) Live Load Coefficient of Variation and (c) Design Parameter $k$