Track Stiffness Measurement with Implementation to Rail/Vehicle Dynamic Simulation

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ABSTRACT

Computer simulation provides an advanced tool for railroads to make vehicle and track maintenance assessments in a timely and more economically strategic manner. However, the validity of a simulation is limited by the information it incorporates. A good simulation of a vehicle and rail structure requires knowledge of the physical characteristics of both the vehicle and track. Some of these physical characteristics can be generated with information such as track geometry car data. However, almost all previous simulations did not incorporate an accurate representation of the vertical elasticity of the track structure, resulting in some loss of accuracy. This paper presents the use of vertical track stiffness measurements along with corresponding track geometry car data implemented into the track/railcar vehicle dynamics software VAMPIRE™. Vertical track stiffness measurements were obtained from a high-speed measurement railcar developed by the University of Nebraska – Lincoln. Simulations are shown that incorporate track geometry car data and measurements of vertical track stiffness over an at-grade road crossing and a set of switches. The simulation results suggest that vehicle dynamics performance can be significantly changed with the inclusion of measured vertical track stiffness data, as compared to assumed nominal track stiffness. Results indicate that the inclusion of vertical track stiffness measurements provides a more accurate computer simulation of track and railcar interaction.
INTRODUCTION

Track structure quality is a major factor in the safety and longevity of railroad operation and equipment. Rail and its support structure are subjected to high repeated forces which promote degradation. Increasing demands have called for higher traffic and the necessary ability to support the increase in traffic. So, in order to keep up with the demand and avoid excessive costs; fast, reliable, and economically strategic methods are needed to locate and prioritize track in need of maintenance in order to minimize delays, avoid derailments, and apply maintenance costs in the most prudent way.

Computer simulation of the dynamic interaction of rail structure and rail vehicles is continuously growing as a viable method to achieve this goal. However, these simulations can be quite limited in their accuracy if they have limited input information. In order to accurately represent a physical system, a simulation must account for the excitation, elasticity, inertia, and dissipation involved with the system. The measurement and representation of these factors as they pertain to rail vehicles is well established, particularly with vehicle springs, masses, and dampers. Conversely, the physical representation of the track structure in simulation is quite incomplete. Most notably absent is the variable vertical elasticity/stiffness associated with track structures.

Geometric factors such as gage, curvature, and crosslevel obtained from track geometry vehicles have largely been the only quantifications of track structure characteristics used in simulation. Such measurements are only the beginning to a complete physical picture of the rail structure. As a result, dynamic simulations of vehicle and rail interaction can be limited. Fortunately, measurement and calculation
techniques are being explored and implemented that can absolve this issue. Presented here is a discussion of such a technique.

**QUANTIFYING TRACK STRUCTURE STIFFNESS**

Many theoretical models have been proposed which can quantify the stiffness of track structures. One thing that they all have in common is that they characterize track stiffness as a function of deflection under load. Two such models are explored; the Cubic Model and the Winkler Model.

Both of these models characterize the track as a Bernoulli beam resting on an elastic foundation. The difference between the two models is in their characterization of the elastic foundation on which the Bernoulli beam rests. This elastic foundation is of course the track substructure composed of ties, ballast, sub-ballast, and sub-grade.

The load deflection characteristic of the track substructure is described by a quantity known as track modulus. Track modulus is defined as the coefficient of proportionality between the rail deflection and the vertical contact pressure between the rail base and track foundation \( (1) \). The Cubic Model proposes two modulus terms, one of which is associated linearly with deflection and the other which is non-linearly associated with deflection. The Winkler Model proposes a single modulus term associated linearly with deflection. The next sections will present the Winkler and Cubic Models which can be solved and then used to calculate track stiffness.
Cubic Model

The Cubic Model represents the spring like reaction of the structure supporting the rail when the rail is deflected under load as,

\[ p(x) = u_1 y(x) + u_3 y(x)^3 \]  

(1)

Where, \( p(x) \) is the reaction of the track substructure, \( u_1 \) and \( u_3 \) are the modulus terms, and \( y(x) \) is the vertical deflection of the rail/substructure. The controlling equation based on a Bernoulli beam on an elastic foundation under load that incorporates the above cubic relation can be shown to be,

\[ EI \frac{d^4 y(x)}{dx^4} + u_1 y(x) + u_3 y(x)^3 = q(x) \]  

(2)

Where, the first term is the Bernoulli beam reaction to loading, the second two terms are the sub-structure reaction to loading/deflection based on the Cubic Model given in Equation 1, and the term \( q(x) \) describes the loading on top of the rail from the wheel.

To solve this fourth order differential equation, four boundary conditions are required. These conditions can be deduced from a free body diagram of the loaded Bernoulli beam and substructure. Once properly posed, the BVP may be solved for given modulus values and a given wheel load using any capable numerical solver.

Winkler Model

The Winkler model describes the deflection of a beam resting on a continuous, uniform elastic foundation similarly to the Cubic Model; however in this model the deflection of the substructure under an applied load is linearly proportional to modulus. That is,
\[ p(x) = uy(x) \] (3)

Where, \( p(x) \) is the reaction of the track substructure when deflected, \( u \) is the modulus value of the support structure and \( y(x) \) is the vertical deflection of the rail/substructure.

This model has been shown to be an effective method for determining track modulus (2).

The Winkler model has a closed form solution described by the equation (2):

\[ y(x) = \frac{-P\beta}{2u} e^{-\beta x} \left[ \cos(\beta x) + \sin(\beta x) \right] \] (4)

where:

\[ \beta = \left( \frac{u}{4EI} \right)^{1/4} \] (5)

where:

- \( P \) is the load applied to the rail.
- \( u \) is the track modulus.
- \( E \) is the modulus of elasticity of the rail.
- \( I \) is the second moment of inertia of the rail.
- \( y(x) \) is the vertical deflection of the rail.
- \( x \) is the distance between the applied load and measurement point.

**Measuring Track Deflection under Load and Calculating Track Stiffness**

Now that the mathematical models for the rail vertical deflection under loading have been developed, a method for calculating track stiffness will be presented which incorporates both the mathematical models and measurement data. This overall vertical track
stiffness can be related to deflection at the wheel/rail interface through the following familiar relationship:

\[
K_E = \frac{P}{y_{\text{wheel}}}
\]  \hspace{1cm} (6)

Where:

- \( K_E \) = Effective Stiffness of Rail and Support Structure
- \( P \) = Load on the rail imposed through the wheel/rail contact point
- \( y_{\text{wheel}} \) = Total vertical deflection of the rail at the wheel/rail contact point.

One method to obtain \( y_{\text{wheel}} \) is through an onboard, non-contact measurement system. Figure 1, (3), depicts the general arrangement of such a measurement system with respect to a loaded train wheel/rail as well as the geometric quantities used to calculate track structure stiffness with this method. This system is based on the measurement system under development at the University of Nebraska-Lincoln by Dr. Shane Farritor (3). This system takes measurements of track deflection adjacent to a loaded hopper car wheel as the car travels at full track speed. In this manner, large expanses of track can be measured and characterized in a reasonable time frame.

Figure 1 shows the geometry and quantities of interest necessary to obtain track stiffness, \( K_E \), with this particular measurement system. In the figure, “H” is ideally a static quantity. It is the constant vertical distance between the measurement sensor plane and the plane of the contact point of the wheel and rail. The quantity “h” is the measurement obtained by the sensor.
Two important relations can be derived from Figure 1 that, along with the implementation of the previously discussed mathematical models of the rail and curve fitting, will allow the measured quantity, “h”, to be related to track stiffness. These two relations are as follows:

\[ y_r = H - h \]  \hspace{1cm} (7)

\[ y_r = y_{sensor} - y_{wheel} \]  \hspace{1cm} (8)

where:

- \( h \) is the vertical distance between the sensor plane and the rail head obtained by system.
- \( H \) is the vertical distance between the sensor system and the wheel/rail contact point.
- \( y_r \) is the relative vertical rail displacement with respect to the wheel/rail contact plane.
- \( y_{sensor} \) is the vertical rail deflection at the position of the sensor measurement.
- \( y_{wheel} \) is the vertical rail deflection at the position of the wheel/rail contact.

Using a mathematical model of rail deflection such as that of the Cubic or Winkler Models discussed previously, one can obtain a vertical deflection profile of the rail as a function of distance from the loading point. The loading point in question is the wheel/rail contact point and the load causing the deflection is the force imposed on the rail through the wheel at the wheel/rail contact point. In this case, however, two loads influence the measured deflection. These loads are those imposed by the two wheels nearest the sensor system. By superposition, the effects of these two loads on the rail can be quantified.
Using superposition and a mathematical model, one can calculate $y_{\text{sensor}}$ and $y_{\text{wheel}}$ and then $y_r$ from equation 8. Doing this repeatedly for varying modulus values ranging from very low to very high and employing equation 6, one can construct an array of track stiffness, $K_E$, and a corresponding array of $y_r$ over the range of modulus values. Then, plotting $y_r$ vs. $K_E$ with $y_r$ as the independent variable, a curve fit can be used to obtain a theoretical relationship between $y_r$ and $K_E$ based on the chosen mathematical model. Finally, given “h” from measured data and using equation 7 to solve for $y_r$, the curve fitted theoretical relationship can then be used to calculate $K_E$ at a measured point along the track.

Through this algorithm, a large amount of deflection measurements can be taken along the track and converted to track structure stiffness. Armed now with the vertical stiffness, $K_E$, in distance history form, a track model incorporating variable vertical stiffness can be created and implemented.

**Track Stiffness Profiles from Winkler and Cubic Models**

Following the procedure outlined above, the plotted relationships of track structure stiffness to “$y_r$” for the Cubic and Winkler Models are shown in Figure 2. As shown in Figure 2 the two models are similar. The cubic model is somewhat flatter and scaled down than that of the Winkler. However, either the Cubic or Winkler model will lead to a reasonable calculation for track stiffness which may then be implemented into a track model for simulation.
SIMULATIONS OF A STANDARD COAL HOPPER CAR

In a typical simulation, only the information obtained from the geometry vehicle is implemented to represent track structure. A typical geometry vehicle can not supply information regarding track stiffness. For this reason, a nominal constant value is used to represent track stiffness. This, of course, cannot accurately account for the variable vertical force input to the vehicle from the track due to changing stiffness conditions. The following sections will show the difference in results for simulations of a typical coal hopper car using only information from a geometry vehicle and constant track structure stiffness, and simulations integrating the variable track stiffness information from the system of the University of Nebraska Lincoln research group described previously.

Simulations with and without Variable Track Stiffness Information

For the following simulation data, VAMPIRE™ vehicle dynamic simulation software was employed. Track stiffness information was gathered and provided by the FRA sponsored team from the University of Nebraska-Lincoln under the supervision of Dr. Shane Farritor. All track geometry was provided courtesy of Dwight Clark and Bill Gemeiner with Union Pacific Railroad.

A few specific points along the track were examined that are widely known amongst railroaders to produce “rough ride” due to a localized change in track stiffness. These are road crossings and switches. The simulation output that will be examined from these simulations is vertical wheel loading.

The first simulated location was a set of switch crossings. The first switch is the transition to a siding and a short distance after is the switch that transitions back from the
siding to the main line. As already stated it is commonly known that passing over switches can and often does create rough ride/high acceleration situations. Shown in Figure 3 is the vertical force on the rail of the left wheel of the first axle from two different simulation analyses. In green are the results of the standard analysis which uses geometry information and constant track stiffness. In red are the results of an analysis incorporating measured variable track stiffness computed with the Winkler model.

The green line of Figure 3 representing the results of a constant stiffness standard simulation analysis is blind to the transition across the switches. A standard analysis based on these results could falsely report this section of track as being of no concern. However, the analysis incorporating the variable stiffness measurements shows clearly the effects of crossing switches. This is seen in the two transient spikes. The first at approximately 700 feet represents the first switch and the second major spike about 2500 feet beyond represents the second switch. This distance between spikes coincides perfectly with the actual track distance between these two switches. With the variable track stiffness information the analysis accounts for the effect of switch crossings on the vehicle while the constant stiffness simulation does not.

Figure 4 is a closer view of the first switch with all three stiffness models. In Figure 4 it can clearly be seen that the variable stiffness simulations show a vehicle response to crossing a switch as would be expected. The constant stiffness simulation shows hardly any response here at all.

As previously stated, the second location investigated was a road crossing. Shown in Figure 5 are the results of the simulation using constant stiffness and the results
of the simulations incorporating variable stiffness from the Winkler and Cubic Models over the road crossing.

As is evident in Figure 5 the two distinct spikes occur at the beginning and end of the road crossing for the simulations with variable stiffness only. This is expected, as transitions from regular subgrade to road surface and back again are often rough due to the immediate change in support stiffness. The standard simulation shown in blue is blind to this effect.

The geometry information variables from the track geometry vehicle which went into the simulations over the road crossing and switches are quite consistent and are well within acceptable limits. Therefore, the road and switch crossings do not show up at all as exceptions or areas of large variation in the standard constant stiffness simulations. Only when using variable stiffness does the presence of a road or switch crossing become evident. This shows very strongly the importance of using variable track stiffness in a simulation in order to achieve greater accuracy. Road crossings and switches are commonly known as areas of rough transition and the inclusion of the track stiffness information reveals this. The standard simulation does not.

**CONCLUSIONS**

First and foremost, in order to have more accurate and informative simulations the actual vertical track stiffness must be included. Failure to represent it limits the ability to predict actual vehicle/track interaction.

Following this logic further, the calculation of track stiffness must be accurate if its effects in simulation are to be accurate. The above models require knowledge of the
load applied to the rail where the deflection is measured. However, the measurement system that was used to gather the deflection measurements makes an assumption of constant vertical wheel loading. As was shown in the simulation results, vertical wheel loading is anything but constant. Because of this, it seems reasonable that knowing the exact load on the measurement wheels while taking deflection measurements would be necessary to accurately calculate track stiffness from deflection measurement. This feature is currently being applied to the UNL system.

Finally, further knowledge about the exact nature of the modulus terms is necessary. More empirical data is required in order to accurately characterize the nonlinearity associated with the ballast and its sub components. This not only applies to the elastic nature of the track sub structure, but its dissipative characteristics as well. Knowledge of these characteristics is largely incomplete at this point, so further investigation is necessary for the evolution of simulation as an accurate and instructive tool.
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REFERENCES


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