AC VERSUS DC

STRAY CURRENT CORROSION, ANALYSIS AND MEASUREMENT

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PART 1  CORROSION DUE TO AC AND DC SIGNALS

1.1  INTRODUCTION

Stray currents in a rail transit environment may consist of direct current components associated with the operation of dc electric traction systems and alternating current components associated with the operation of ac electric traction systems and electric utility systems. The impact on the infrastructure will be different for the unidirectional flow of iron ions from a structural steel element due to dc stray currents than for the flow of iron ions from a structural steel element on one half-cycle and the reversal of current into the structural steel element on the other half-cycle due to ac stray currents. The consideration of this issue will yield functional and economic benefits in designing stray current mitigation measures that are commensurate with the electrical environment.

1.2  FARADAY’S LAWS OF ELECTROLYSIS

Michael Faraday (1791-1867) conducted experimental research involving the relationship between chemistry and electric current and published his results in June of 1833 in a paper entitled On Electro-chemical Decomposition. The following are among the significant findings that emerged from his research:

- The mass of a substance altered at an electrode during electrolysis is directly proportional to the quantity of electric charge transferred at that electrode.
- The mass of an elemental material altered at an electrode due to the quantity of electric charge transferred at that electrode is directly proportional to the equivalent weight of the elemental material.

The equivalent weight of an elemental material is a measure of the mass of the material involved in an electrochemical reaction and the amount of electric charge transferred due to the passage of an electric current. This principle is discussed in detail in the following section.

1.3  REVIEW OF CHEMICAL PRINCIPLES

A review of some principles of chemistry will assist in our analysis. The nucleus of an atom contains protons and neutrons. An element is uniquely defined by the atomic number which is the number of protons in the nucleus. The mass number is the total number of protons and neutrons in the nucleus. Whereas the number of protons in the nucleus of an atom of a given element does not change, the number of neutrons in the nucleus of an atom may vary. An isotope is an atom of a given element with a specific number of neutrons. The most common isotope of carbon is carbon-12 which contains 6 protons and 6 neutrons and hence has a mass number of 12. This is the basis of the definition of the mole which is the number of atoms in 12 grams of carbon-12 or $6.022 \times 10^{23}$ atoms. The molar mass is the mass of one mole of a substance. The molar mass of carbon-12 is then 12 grams. We will be concerned with the molar mass of iron in this analysis since much of corrosion control in the rail transit industry deals with the impact of stray currents from the electric traction system on iron-based components of the infrastructure such as water and gas mains, structural steel, and the steel
reinforcement of concrete structures. Iron (chemical symbol \( Fe \)) has an **atomic number** of 26. The **molar mass** of iron is 55.845 grams per mole. The most common isotope of iron has a mass number of 56 which includes 26 protons and 30 neutrons.

Electrons have a negative electrical charge, protons have a positive electrical charge and neutrons have zero electrical charge. Electrons and protons have equal and opposite electrical charges. A neutral atom has the same number of electrons and protons and hence exhibits zero electrical charge. An **ion** is an atom from which one or more electrons have been removed or added. The addition of electrons results in the reduction of electrical charge to a more negative charge and is hence referred to as a reduction reaction. Conversely, the removal of electrons results in the increase of electrical charge to a more positive charge and is referred to as an oxidation reaction since oxygen was originally thought to be the only substance that could cause such a reaction. The most common ions of iron are \( Fe^{+2} \) (ferrous) and \( Fe^{+3} \) (ferric). In a typical corrosion reaction involving iron in an aqueous environment such as moist or wet soil, two electrons are removed to form the ferrous ion \( Fe^{+2} \). Water in the soil (\( H_2O \)) disassociates into hydrogen ions (\( H^+ \)) and hydroxyl ions (\( OH^- \)). The reactions that occur are as follows:

\[
Fe(s) \rightarrow Fe^{+2}(aq) + 2e^- \\
2H^+(aq) + 2e^- \rightarrow H_2(g) \\
Fe^{+2}(aq) + 2OH^- \rightarrow Fe(OH)_2
\]

The end product \( Fe(OH)_2 \) is ferrous hydroxide which is a form of what is commonly referred to as rust. Subsequent reactions may yield \( Fe_2(OH)_3 \) or ferric hydroxide.

### 1.4 THE ELECTRICAL DECOMPOSITION OF IRON

#### 1.4.1 Constant Current Case

We are now prepared to evaluate the electrical decomposition of iron. Based on the information contained in this section, the equivalent weight can now be defined more specifically as the mass of a given substance that will react with one mole of electrons in an oxidation-reduction reaction. For the first ionization state of iron from a neutral atom \( Fe \) to a ferrous ion \( Fe^{+2} \), the equivalent weight is obtained from the molar mass \( M \) of 55.845 grams per mole and the number of moles \( z \) of electrons transferred in the reaction.

\[
\frac{M}{z} = \frac{55.845}{2} = 27.923 g
\]

The equivalent weight of 27.9225 g is therefore the mass of iron removed from the metallic lattice of the substrate for each mole of electrons transferred in the oxidation-reduction reaction.

Faraday’s law can be summarized as follows:

\[
m = \frac{Q}{F} \left( \frac{M}{z} \right)
\]
Where

\( m \) is the mass of substance liberated in grams (g)

\( Q \) is the total charge transferred in the reaction in Coulombs (C)

\( F \) is the Faraday constant 96,485 Coulombs per mole of electrons (C-mol\(^{-1}\))

\( M \) is the molar mass of the substance in grams per mole (g-mol\(^{-1}\))

\( z \) is the number of moles of electrons transferred per number of moles of substrate

We can now calculate the mass of iron altered due to the flow of one ampere of stray current for a year which is typically cited as a measure of concern for electrified transit systems. One ampere is equivalent to the flow of 1 Coulomb of charge per second.

\[
I = \frac{Q}{t} = \frac{1 \text{C}}{1 \text{s}} = 1 \text{A}
\]

There are 31,536,000 seconds in a year. The amount of charge transferred in a year due to 1 ampere of stray current is

\[
Q = It = \left(1 \frac{\text{C}}{\text{s}}\right)(31,536,000 \text{s}) = 31,536,000 \text{C}
\]

The number of moles of electrons transferred in the reaction can be obtained from the total charge transferred and the Faraday constant.

\[
\frac{Q}{F} = \frac{31536000}{96,485} = 327 \text{moles}
\]

The equivalent weight cited previously is

\[
\frac{M}{z} = 27.923 \text{g}
\]

The mass of iron consumed by the passage of 327 moles of electrons and an equivalent weight of 27.923 grams per mole of electrons is

\[
m = \left(\frac{Q}{F}\right)\left(\frac{M}{z}\right) = (327)(27.923) = 9,131 \text{g}
\]

The conversion of 9.131 kilograms to pounds at a unit conversion of approximately 2.2 pound per kilogram yields 20 pounds of iron.

\[
W = 9.131 \text{kg} \cdot 2.2 \frac{\text{lb}}{\text{kg}} \approx 20 \text{lb}
\]

In summary, a constant current of 1 ampere for a period of 1 year will consume 20 pounds of iron.
1.4.2 Time-varying Current Case

The Faraday constant $F$, molar mass $M$, and valence number $z$ are constant for a given reaction. The mass of substance liberated in the reaction therefore varies as the total charge $Q$ transferred in the reaction. The total charge transferred over the period of the reaction may be given by

$$Q = \int_0^t I \, d\tau$$

Where

- $Q$ is the total charge transferred in Coulombs (C)
- $I$ is the current as a function of time $\tau$
- $t$ is the period of the reaction

It was noted previously that the corrosion of iron is characterized by the flow of current from the substrate as iron ions. It is therefore theoretically possible to prevent the corrosion of iron by causing current to flow into the iron throughout the entire surface. This principle is referred to as cathodic protection where the iron becomes the cathode in an electrochemical cell. In this case, a sacrificial anode such as a magnesium or zinc anode which is less noble than iron corrodes sacrificially or an impressed current anode such as a graphite electrode discharges current from a dc power source through the soil or other medium into the iron substrate thus providing protection from electrolytic corrosion. Let us consider a sinusoidal current over the period of 1 cycle.

$$Q = \int_0^t \sin \omega \tau \, d\tau$$

$$= \frac{1}{\omega} \int_0^{2\pi} \sin \omega \tau \omega d\tau$$

$$= \frac{1}{\omega} \cos \omega t \left[ \frac{2\pi}{0} \right]$$

$$= \frac{1}{\omega} [ \cos(2\pi) - \cos(0) ]$$

$$= \frac{1}{\omega} [ 1 - 1 ]$$

$$= 0$$

The total charge $Q$ transferred in the electrochemical reaction due to a sinusoidal stray current is zero. It can therefore be considered that in theory an ac stray current with equal positive and negative half cycles will support the corrosion of the iron on the positive half-cycle and will protect the iron on the negative half-cycle. It should be noted, however, that corrosion will occur under the circumstance that not all of the iron plated off on the positive half-cycle is plated back on the negative half-cycle.
PART 2   ANALYSES OF PERIODIC WAVEFORMS

2.1  FOURIER ANALYSIS

Joseph Fourier (1768-1830) conducted research regarding the mathematical theory of heat conduction. He published his memoir *On the Propagation of Heat in Solid Bodies* in 1807 and his essay *The Analytic Theory of Heat* in 1822. Fourier found in conjunction with his research that the mathematics of heat conduction can be modeled as a series of sinusoidal waves. His work was later applied to the representation of periodic waveforms into series of sine and cosine waves of various amplitudes and frequencies. While a formal discussion of the Fourier series is beyond the scope of this text, the following are among the significant issues that relate to our work in stray current analysis.

2.2  MATHEMATICAL RELATIONSHIPS

The Fourier series for a periodic waveform which we will describe as a function of time $f(t)$ may be represented as the sum of a constant term and a series of sine waves and cosine waves of various amplitudes and frequencies.

$$f(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \ldots + a_n \cos n\omega_0 t + \ldots + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \ldots + b_n \sin n\omega_0 t + \ldots$$

Where

$n$ is the number of the component harmonic

$\omega_0$ is the fundamental radian frequency of $f(t)$.

As an example, we will construct a periodic waveform that is the sum of a dc component, a 25-Hz ac component, and a 60-Hz ac component with amplitudes of 1 volt and construct the corresponding amplitude and phase spectra. The periods associated with the 25-Hz and 60-Hz components are

$$T_{25} = \frac{1}{f} = \frac{1}{25} = 0.0400\text{s}$$

$$T_{60} = \frac{1}{f} = \frac{1}{60} = 0.0167\text{s}$$

The periods of the component sine waves are therefore 40 milliseconds for 25-Hz and 16½ milliseconds for 60 Hz. The sum of the two waveforms yields a periodic waveform that repeats over the least common multiple of 40 milliseconds and 16½ milliseconds which is 200 milliseconds. This is the fundamental period of the total waveform which yields a fundamental frequency of

$$f = \frac{1}{T} = \frac{1}{0.2} = 5\text{Hz}$$
The harmonic components represented by the 25-Hz and 60-Hz signals are then

\[
\frac{25\text{Hz}}{5\text{Hz}} = 5, \quad \frac{60\text{Hz}}{5\text{Hz}} = 12
\]

The 25-Hz frequency is then the 5th harmonic of the fundamental frequency and the 60-Hz frequency is the 12th harmonic frequency. In the next section, we will demonstrate the synthesis of the summation of a 0-Hz dc electric traction component, a 25-Hz sinusoidal electric traction frequency and a 60-Hz sinusoidal electric utility frequency to produce a composite waveform and the recovery of those frequencies via Discrete Fourier analysis of actual waveforms obtained from field measurements. The dc, 25-Hz and 60-Hz waveforms are shown in Figures 1, 2 and 3. The composite waveform which is the sum of the dc component and the 25-Hz and 60-Hz waveforms is shown in Figure 4.
2.3 AMPLITUDE AND PHASE SPECTRA

The periodic waveform and the component harmonic frequencies are represented below in the time domain.

\[
f(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \ldots + a_n \cos n\omega_0 t + \ldots + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \ldots b_n \sin n\omega_0 t + \ldots
\]

A more convenient representation may be provided for our purposes, however, in the frequency domain by taking advantage of the following relationship:

\[
a_n \cos n\omega_0 t + b_n \sin n\omega_0 t = c_n \cos n\omega_0 t + \theta_n
\]
Where

\[ c_n = \sqrt{a_n^2 + b_n^2} \quad \text{and} \quad \theta_n = -\tan^{-1} \frac{b_n}{a_n} \]

This format permits us to represent the component frequencies via the amplitude spectrum which indicates the amplitude of each component frequency and a phase spectrum which indicates the phase angle of each component phasor.

### 2.3.1 AMPLITUDE SPECTRA

The amplitude variables \( c_n \) are calculated on the basis that the amplitudes or peak values for the 0\(^{th}\) harmonic (dc), 5\(^{th}\) harmonic (25 Hz), and 12\(^{th}\) harmonic (60 Hz) are equal to 1 V.

\[ c_n = \sqrt{a_n^2 + b_n^2} = \sqrt{1^2 + 0^2} = 1 \]

The amplitude spectrum shown in Figure 5 indicates the 0\(^{th}\) (dc) component and the 5\(^{th}\) harmonic component (25-Hz) and 12\(^{th}\) harmonic component (60-Hz) sinusoids identified in the composite waveform.

![Amplitude Spectrum](image)

Figure 5 Amplitude Spectrum

### 2.3.2 PHASE SPECTRA

A note regarding the phase spectra is in order. The flow of harmonic currents through an RLC network will yield voltages of different phase angles for each harmonic frequency component. The results of a Fourier analysis will therefore yield complex numbers for each harmonic frequency component.

\[ c_n = a_n + jb_n \]

The phase angle is in turn calculated from the coefficients of the sinusoidal and cosinusoidal coefficients.
\[ \theta_n = -\tan^{-1} \frac{b_n}{a_n} \]

The waveform synthesized in this analysis consists only of sine waves which would result from the flow of harmonic currents through a resistive network where inductance and capacitance which would result in phase angle differences for each frequency component are not present. Since cosine waves are not present in the composite waveform and the \( a_n \) coefficients are equal to zero, the phase angle for each harmonic frequency component would be undefined due to division by zero. It can be seen, however, that the phase variables \( \theta_n \) approach -90 degrees in the limit as the \( a_n \) coefficients approach zero. The phase spectrum is shown in Fig. 6.

\[
\lim_{a_n \to 0} \left[ -\tan^{-1} \frac{b_n}{a_n} \right] = -90^\circ
\]

![Phase Spectrum](image)

**Figure 6 Phase Spectrum**

### 2.4 DISCRETE FOURIER TRANSFORM

The Discrete Fourier Transform (DFT) is based on the principle that multiplying a composite waveform by frequencies that are contained in that waveform yields the component frequencies whereas multiplying the composite waveform by frequencies that are not contained in that waveform produces a zero result. In this manner, it is possible to identify the magnitude and phase of each component frequency of a composite waveform. The tabulations that follow in Table 1 are the result of the DFT calculations on the composite waveform that was synthesized in the prior section which includes the 25-Hz and 60-Hz component frequencies as shown in the amplitude spectrum of Figure 1.

The term “band-limited” refers to the condition in which a finite number of frequencies are contained in the composite waveform. Since we have constructed the waveform from a 25-Hz frequency with unit amplitude and a 60-Hz frequency of unit amplitude, we already know the frequencies that are contained in the waveform. We have already determined that the fundamental frequency of the composite waveform is 5 Hz with a period of 200 milliseconds.
The following points are noted with respect to the data in Table 1:

- Column 1 represents the sample time ‘t’ which extends from 0 to 200 milliseconds.
- Column 2 represents the 25-Hz frequency \( f_1(t) \) which is the 5th harmonic of 5 Hz.
- Column 3 represents the 60-Hz frequency \( f_2(t) \) which is the 12th harmonic of 5 Hz.
- Column 4 represents the composite waveform \( f(t) \) which is the fundamental frequency of 5 Hz.
- Columns 5 through 16 contain the products of the composite waveform and the harmonics through the 12th harmonic which we know to be the limit of the component frequencies.
- The last row of the table represents the summation of the products of the composite waveform and the harmonic frequencies through the 12th harmonic.

The last row of the table reveals the unit amplitude of the 0th harmonic (dc), the 5th harmonic (25 Hz) and the 12th harmonic (60 Hz) with zero values for harmonics other than the 5th and 12th harmonic.

In Part 3 Measurements of Periodic Waveforms we will demonstrate the need to sample at a rate that will permit the identification of dc versus ac components in the waveform which will be an important consideration in the determination of the required stray current mitigation measures. An observation of the 25-Hz waveform shown in Fig. 2, for example, indicates that a sampling period \( \Delta t \) that is equal to the period \( \tau = 0.04 \) s of the 25-Hz waveform beginning at \( t = 0 \) would yield an amplitude of zero for each sample which would incorrectly indicate that a 25-Hz component is not present. A sampling period of \( 2 \Delta t \) beginning at \( t = 0 \) would yield the same result. The Nyquist Criterion and subsequent sampling theory considerations require that a sample rate of greater than twice the frequency of the highest-frequency harmonic contained in the waveform is required for accurate identification of the harmonic components.
<table>
<thead>
<tr>
<th>Time</th>
<th>Frequency Components</th>
<th>Composite</th>
<th>$\sin(n\omega_0t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>t(s)</td>
<td>$f_1(t) = \sin(2\pi<em>25</em>t)$</td>
<td>$f_2(t)=\sin(2\pi<em>60</em>t)$</td>
<td>$f(t)=f_1(t)+f_2(t)$</td>
</tr>
<tr>
<td>0.000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.001</td>
<td>0.1564</td>
<td>0.3681</td>
<td>1.525</td>
</tr>
<tr>
<td>0.002</td>
<td>0.3090</td>
<td>0.6845</td>
<td>1.994</td>
</tr>
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<td>0.003</td>
<td>0.4540</td>
<td>0.9048</td>
<td>2.359</td>
</tr>
<tr>
<td>0.004</td>
<td>0.5878</td>
<td>0.9980</td>
<td>2.586</td>
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<td>...</td>
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</tr>
<tr>
<td>0.196</td>
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<td>-0.9980</td>
<td>-0.586</td>
</tr>
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<td>0.200</td>
<td>0.0000</td>
<td>0.0000</td>
<td>1.000</td>
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</table>
Table 2  Composite Waveform Components (DC and 25 Hz and 60 Hz)

<table>
<thead>
<tr>
<th>Time (t(s))</th>
<th>Frequency Components</th>
<th>Composite</th>
<th>$2/\tau\sum f(t)\sin(\omega_0 t)\Delta t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1(t) = \sin(2\pi<em>25</em>t)$</td>
<td>$f_2(t)=\sin(2\pi<em>60</em>t)$</td>
<td>$f(t)=f_0(t)+f_1(t)+f_2(t)$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>0.001</td>
<td>0.156</td>
<td>0.368</td>
<td>1.525</td>
</tr>
<tr>
<td>0.002</td>
<td>0.309</td>
<td>0.685</td>
<td>1.994</td>
</tr>
<tr>
<td>0.003</td>
<td>0.454</td>
<td>0.905</td>
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<td>0.588</td>
<td>0.998</td>
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<tr>
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<td>0.199</td>
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<td>-0.368</td>
<td>-0.525</td>
</tr>
<tr>
<td>0.200</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Amplitude: 0 0 0 0 1 0 0 0 0 0 0 1
Phase: 0 0 0 0 -90 0 0 0 0 0 0 -90
Frequency: 5 10 15 20 25 30 35 40 45 50 55 60
Harmonic: 1 2 3 4 5 6 7 8 9 10 11 12
PART 3 MEASUREMENTS OF PERIODIC WAVEFORMS

3.1 MEASURED SIGNAL COMPOSITION: DISCRETE FFT

The following voltage measurements were taken in soil via two test electrodes. The waveform was recorded at a rate of 1,000 samples per second for 1 second (1,000 samples). First, the recorded raw data was imported from the measuring instrument.

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<th>-79.8</th>
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</thead>
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<td>1.186553·10^6</td>
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<td>1.186554·10^6</td>
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<td>f</td>
<td>1.186556·10^6</td>
<td>-141</td>
</tr>
<tr>
<td>f</td>
<td>1.186557·10^6</td>
<td>...</td>
</tr>
</tbody>
</table>

The first column represents the sample number while the second column represents the electrical potential difference in millivolts between the two test electrodes in the soil. Next, a Discrete Fourier Transform (DFT) was performed on the waveform and the absolute value was plotted from 0 to 499 Hz: this is the maximum signal frequency that can be accurately identified given a 1000 Hz sampling rate according to the Nyquist Criterion.

Figure 5 DFT of Measured Waveform 0 Hz to 499 Hz
The plots in Figures 5 and 6 above show that the DFT analysis identified the magnitude and frequency of the ac signals in the waveform, including harmonics, as well as the magnitude of dc component. In this example, a dc component of approximately 80 mV was obtained from the recorded data. It should be noted that the dc value was determined as the average value of the waveform. Also, 25 Hz and 60 Hz (including its third harmonic of 180 Hz) signals were identified.

3.2 EFFECT OF SAMPLING RATE

Sampling is the process of converting a continuous waveform into a discrete set of values. Selection of a proper sampling rate is paramount prior to waveform measurement. The Nyquist Criterion states that in order to reconstruct a band limited waveform, the sampling rate must be greater than twice the highest frequency of that waveform. It was mentioned that the sampling rate chosen was 1000 samples per second in the example above. The highest frequency in the soil sometimes cannot be known ahead of time and engineering judgment must be made. Additionally, higher frequency harmonics in the soil must be considered. Therefore, 1000 samples per second was an adequate sampling rate since the highest frequency expected in the soil was the third harmonic of the public utility 60 Hz signal, or 180 Hz, and the frequencies that can be accurately identified with this sampling rate according to the Nyquist Criterion are all frequencies less than 500 Hz. A safety margin was included in the selection of the 1000 samples per second sampling rate for unexpected frequencies higher than 180 Hz.

In order to show the effect of an inadequate sampling rate, the same measured data was taken at 1 sample per second for 1000 seconds (1000 samples). Again the measured data was analyzed and a DFT was performed and plotted.
A dc component of approximately 88 mV was obtained from the recorded data. The dc component is likely to be inaccurate due to the low sampling rate, which causes the under sampled ac signals in the waveform to potentially skew the result. Additionally, other signal frequencies in the soil cannot be accurately identified, in this case, 25 Hz and 60 Hz (as well as its third harmonic of 180 Hz).

**PART 4 CONCLUSION**

The relative contributions of the ac and dc components of stray currents will have a direct bearing on the extent of the required mitigations measures. It was shown in Part 1 that a constant dc or unidirectional current, which may be due to the operation of a dc transit system, that results in a net flow of charge and hence a net alteration of the metal substrate will require a greater degree of mitigation than a sinusoidal ac current of the same effective magnitude that may be due to electric utility ground and/or ac traction currents. The ac and dc components may be evaluated via the Discrete Fourier Analysis of the measured waveform with the caveat that the analysis of the highest-order harmonic is dependent on the sampling frequency. It was also noted that the dc component may be obtained as the average value of the measured waveform. The sampling rates were demonstrated at 1,000 samples per second and 1 sample per second in order to illustrate the importance of a sampling rate that is adequate for the measurement process. In summary, the cost of mitigation will therefore ultimately depend on the correct measurement technique.
BIBLIOGRAPHY

Fundamental Researches in Electricity, Vol. 1, Michael Faraday, Green Lion Press, Santa Fe, NM, reprinted from the first edition, 1839.