A Simple and Efficient Train Braking Algorithm for PTC Systems

Chinnarao Mokkapati
Robert D. Pascoe
Ansaldo STS USA Inc.
Union Switch & Signal Building
1000 Technology Drive
Pittsburgh, PA 15219

Abstract

This paper outlines a simple algorithm for safe speed enforcement during the braking of freight and passenger trains operating under Positive Train Control System requirements. The train’s on-board PTC subsystem computes the maximum allowed speed for the train during every machine cycle, taking into consideration all speed restrictions the train must obey in a look-ahead distance (e.g., 3 miles) of its current location. It provides continuous supervision to the train crew regarding the distance and time to a penalty brake application. The crew must control the speed of the train to be at or under a maximum allowed speed limit to avoid a penalty brake application. The maximum allowed speed is either the profile velocity when braking to a lower target speed or an existing speed limit, whichever is lower, at the current location of the train. The profile velocity for the train is computed with the help of a simple braking profile algorithm that provides the best possible operational efficiency of the train while not exceeding the safe stopping target location when a penalty brake application is incurred.

1. INTRODUCTION

This paper describes the braking function performed by the On-Board Computer (OBC) of a typical vital Positive Train Control System for safe speed enforcement of passenger and freight trains. Section 2 of the paper presents the basic requirements to be met by the braking function. Section 3 describes the implementation of the braking function with the help of a Braking Profile Algorithm (BPA). Section 4
deals with the derivation of the Braking Parameters used by the BPA. Section 5 describes the validation of the braking function.

1.1 DEFINITIONS And ACRONYMS

The following definitions and acronyms are used in this paper:

**Average Grade** – the average change of elevation between two points along the rails. For the braking function, the average grades between successive target locations within 3 miles in advance of a train’s location are computed and stored in a table of targets. Once the Most Restrictive Target (MRT) is found for the train’s current location, the average grade from the train’s trailing end and the MRT is computed and used in generating the Braking Profile to the MRT.

**Brake Capacity (α)** - the steady-state or maximum brake rate of a given train type (mph/s)

**Brake Build Up Time (T)** – the time duration between braking effort initiation and the achievement of the steady state brake rate.

**Brake Propagation Delay Time (Td)** – the time duration between the brake application request by the On-Board Computer (OBC) and the time that the braking effort begins

**Braking Profile** – a distance / velocity curve that serves as a speed violation threshold resulting in initiation of a penalty brake application when exceeded

**BPA** – Braking Profile Algorithm

**CF** – Correction Factor to account for car parameter variations

**CTC** – Centralized Traffic Control

**DTC** – Direct Traffic Control

**Distance to Target (DTT)** – the distance in feet from the leading end of the train to a targeted location ahead of the train

**E Brake Warning** – an emergency brake warning that is announced to the train engineer via the LDU that occurs during a penalty brake application when the required brake rate is not being satisfied by the penalty brake application

**Enforceable Speed Limit (ESL)** – is the lower of a) the Maximum Authorized Speed (MAS) applicable to the train’s current location or b) the profile velocity that the train must adhere to, in order to avoid a penalty brake application as the train approaches a speed restriction.

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FSB – Full Service Brake

Grade – the elevation at a point location on the railroad

Grade Segment – a section of track with constant grade

HE – Head-End of train

HOT – Head-End of Train – the end of the train where the controlling locomotive is located

LDS – Location Determination System

LDU – Locomotive Display Unit

Maximum Authorized Speed (MAS) – the maximum speed the train could be traveling at on a given segment of track in front of a target

Most Restrictive Target (MRT) – the speed target that must be acted on to ensure that all intervening speed targets within 3 miles of the train’s current location are also satisfied

Multiple Targets – the presence of more than one speed target within 3 miles of the train’s current location

OBC – On-Board Computer

Overspeed Condition – a condition of the train when its speed exceeds its ESL associated with its current location. (Note that a penalty brake action will not be initiated until the train’s speed exceeds the buffered ESL, if a Speed Limit Buffer is in effect.)

Overspeed Protection – an OBC function whereby the OBC protects the train from exceeding its current ESL or buffered ESL if applicable, by requesting a full service brake (penalty brake) application.

Penalty Brake Request – a full service brake request in response to a penalty condition

Penalty Condition – a condition that results from a Profile Curve violation that requires a penalty (full service brake) application. This condition is held active until all affecting conditions have been cleared and the train has come to a complete stop.

Profile Velocity – the train speed at which a penalty brake application will result during a train’s speed reduction to a target, as a function of distance to the target.

PTC – Positive Train Control

Speed Limit Buffer – the buffer added to the ESL or a non-zero speed target, to define the overspeed limit at or above which a penalty brake application will result. As an example, the following speed limit buffers could be used:
For ESL < 20 mph, the buffer is 3 mph
For ESL ≥ 20 mph, the buffer is 5 mph
For a zero (0) speed target, the buffer is zero (0) mph
For a "Restricted" speed limit, the buffer is 3 mph, but the maximum buffered speed shall not exceed 20 mph, (i.e., for a 10R Restricted speed limit, the buffered ESL is 13 mph; for a 15R Restricted speed limit, the buffered ESL is 18 mph; but for a 20R Restricted speed limit, the buffered ESL is 20 mph).

Target Speed – the speed limit, not including the Speed Limit Buffer, associated with a target position

Target Speed Limit (TSL) – the buffered ESL as defined under Speed Limit Buffer

Time to Penalty (TTP) – the time (in seconds) that a train can travel at its current speed before a penalty brake application will occur in response to a penalty braking curve violation

TOES™ – Train Operations and Energy Simulator, developed by Transportation Technology Center, Inc. (TTCI)

Track Map – the representation of the center line of all mapped track, including all critical features. The Track Map is contained in the Track Data Base

Train Length – the length of the entire train consist expressed in feet

Train Speed – the measured speed of the train

Train Weight – the weight of the entire train consist expressed in integer tons.

2. REQUIREMENTS OF TRAIN BRAKING FUNCTION

The braking function of a PTC System must ensure safe braking for all consist variations of all types of trains traveling on a PTC Railroad. For instance, train types could be:

- Freight Car Trains (mix of different types of freight cars, including flat cars, tank cars, box cars, hoppers, air dump cars, gondolas, etc.)
- Unit Hopper Trains (all steel, all aluminum, or a mix of steel and aluminum hoppers)
- Unit Tank Car Trains
• Intermodal Trains (Trailers on Flat Cars)
• Passenger Car Trains (head-end power only or push-pull type)
• Mixed Passenger/Freight Car Trains

In general, the braking function shall ensure and enforce safe train speed, on (or approaching) mapped track, in both forward and reverse directions, under the following and other conditions a railroad may specify:

a) Approaching movement authority limits;
b) Approaching Form A, B and S bulletin limits;
c) Approaching civil speed restrictions;
d) Approaching switches requiring position confirmation;
e) Approaching Form C restrictions;
f) Approaching signals in CTC territory;
g) Approaching controlled track from uncontrolled track;
h) Maintaining speed at or below the relevant speed limit on controlled, mapped/uncontrolled and unmapped/uncontrolled track; and
i) Approaching OS and Track Integrity circuits in DTC territory indicating OCCUPIED.

3. BRAKING FUNCTION IMPLEMENTATION

The braking function is implemented by the OBC. This section describes the key inputs to the braking function, the overall calculations performed by the braking function, and the resulting overspeed protection and braking enforcement process.
3.1 Braking Function Variables

Table 3-1 lists the key variables for the braking function along with their source. These variables are stored in a Local Target Table in the OBC, which serves as a “sliding window” to hold only the targets within 3 miles of the train’s current location in the direction of travel.

<table>
<thead>
<tr>
<th>Input Data</th>
<th>Data Source</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current train location</td>
<td>Location Determination System (LDS) Function</td>
<td>Leading and trailing end-of- train locations are the ‘worst case’ locations</td>
</tr>
<tr>
<td>Current train speed</td>
<td>LDS Function</td>
<td>Adjusted for slip/slide as required</td>
</tr>
<tr>
<td>Current speed limits</td>
<td>OBC Database</td>
<td>Includes all speed restrictions, civil speeds, temporary, and bulletins</td>
</tr>
<tr>
<td>Target location(s)</td>
<td>OBC Database</td>
<td></td>
</tr>
<tr>
<td>Target Speed Limit(s)</td>
<td>OBC Database</td>
<td></td>
</tr>
<tr>
<td>Maximum Authorized Speed (MAS)</td>
<td>OBC Database</td>
<td></td>
</tr>
<tr>
<td>in front of target(s)</td>
<td>OBC Database</td>
<td></td>
</tr>
<tr>
<td>Average grade between targets</td>
<td>OBC Database</td>
<td></td>
</tr>
</tbody>
</table>

3.2 Braking Function Overview

Using the input data for all targets within the Local Target Table 3 mile window, the braking function performs the following calculations which are shown in the step-by-step flowchart in Figure 3-1 and are described in more detail in the following sections:

1. Determine the Most Restrictive Target (MRT) ahead of the train’s current position considering the associated speed restrictions and distances.

2. Calculate the average grade between the train’s trailing end location (worst case) and the selected target.

3. Using the Braking Profile Algorithm, calculate the Profile Velocity to the selected target.

4. Set the Enforceable Speed Limit (ESL) as the lower of a) the civil speed limit or temporary speed restriction applicable to the train’s current location or b) the profile velocity that the train
must adhere to in order to avoid a penalty brake application as the train approaches a speed restriction associated with the selected target.

5. Calculate the Distance to Target (DTT) and Time to Penalty (TTP) associated with the selected target.
Train Initialization (while train is at stop or not on mapped track)
1. Obtain Train Consist parameters from CAD for the given Train Type (Train ID).
2. Compute Braking Parameters $T_d$, $T$, and $\alpha$ for the Train Type, using the pre-determined Braking Parameter Equations.
3. Location Determination System (LDS) initializes train location (establishes direction of travel, initial location on mapped track, etc.)

<table>
<thead>
<tr>
<th>Train movement starts on mapped track</th>
</tr>
</thead>
</table>

During every computation cycle (1 sec):
1. Train’s current location is known from the Location Determination System.
2. Train’s speed $v_a$ (corrected for spin/slide as required) is known from the speed measurement function.
3. Data on all targets within 3 miles ahead of the train’s current location in the direction of travel are loaded into a Local Target Table:
   a. Target Speed Limit (TSL) $v_T$ for each target, knowing all civil speed restrictions, bulletin restrictions, and Maximum Authorized Speed (MAS) for each target.
   b. Distance between successive targets
   c. Average grade over different track fragment groups between targets
   d. Maximum Authorized Speed (MAS) prior to each target (previous target’s speed limit)

Train movement continues at speed under the buffered Enforceable Speed Limit (ESL) and with penalty brake application if buffered ESL is exceeded.

<table>
<thead>
<tr>
<th>Are there any speed restrictions lower than the current ESL within the next 3 miles?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
</tr>
<tr>
<td>No</td>
</tr>
</tbody>
</table>

Determine the Most Restrictive Target (MRT) to profile to, without violating any of the intervening TSLs, among all the targets within 3 miles ahead of the train’s current location.

<table>
<thead>
<tr>
<th>Determine the Most Restrictive Target (MRT) to profile to, without violating any of the intervening TSLs, among all the targets within 3 miles ahead of the train’s current location.</th>
</tr>
</thead>
</table>
| 1. Profile Velocity $v_p$ to MRT is known from previous step.  
2. Determine the ESL for the current location of the train as the Profile Velocity $v_p$ or the current TSL, whichever is lower.  
3. Calculate the Time To Penalty (TTP) using $v_p$, $v_a$, $v_T$ and Distance to Target $S_{DTT}$.  
4. Display Distance To Target (when $DTT \leq 15,000'$), TTP (when $TTP \leq 40$ sec.), and associated warnings to Train Crew. |

Train Engineer controls train speed, keeping it at or under the buffered ESL application and brings train to stop.

<table>
<thead>
<tr>
<th>Train reaches MRT location</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBC calls for Penalty Brake Application and brings train to stop</td>
</tr>
</tbody>
</table>

Figure 3-1. Overall Braking Function Flowchart
**Step 1: Determining the Most Restrictive Target (MRT)**

Using the current local target table, the braking function determines the Most Restrictive Target (MRT) ahead of the train’s current position. The process first compares the two farthest targets by calculating an estimated profile to the farthest target and comparing the associated profile speed at the location of the next closest target with the speed limit at that target. If the profile velocity calculated to the farthest target will exceed the Enforceable Speed Limit (ESL) of the closer target, then the algorithm eliminates the farthest target (as being the most restrictive) and adds the next nearest target for comparison. This process continues until only the MRT remains. Once the MRT is selected, then a profile velocity is calculated to the MRT using the average grade from the worst case location of the trailing end of the train to the MRT.

To illustrate this process, consider the example shown in Figure 3-2 which shows four targets T1, T2, T3, and T4 within 3 miles of the train’s current position. The braking function determines the Distance To Target (DTT) from the current location for each of the four targets, labeled DTT1, DTT2, DTT3, and DTT4 in the figure. These are then ordered and analyzed starting with the furthest target first, which in this example is Target T1 with a 0 mph target speed. The Braking Function calculates Profile P1 to Target T1 and compares the profile to the 15 mph speed limit at Target T2 (the horizontal red line between Targets T1 and T2 in the figure). Since Profile P1 violates the 15 mph speed limit at Target T2 (as well as the other intervening speed limits at Targets T3 and T4), Profile P1 is removed from consideration. The Braking Function then calculates Profile P2 to Target T2 and compares the profile to the 20 mph speed limit at Target T3. Since Profile P2 violates the 20 mph speed limit, Profile P2 is removed from consideration. Now the Braking Function calculates Profile P3 to Target T3 and compares the profile to the 30 mph speed limit at Target T4. Since Profile P3 satisfies the 30 mph restriction, the 20 mph restriction at Target T3 becomes the Most Restrictive Target (MRT). Once the train reaches Target T3, the process is restarted by looking ahead to Target T1 and then Target T2.
(assuming there are no additional targets ahead in the 3 mile look-ahead window). The Braking Function will look at the 0 mph target at Target T1 and generate Profile P4. This profile will satisfy the 15 mph speed limit in front of Target T1 as well as enable the engineer to stop the train at Target T1.

**Figure 3-2. Determination of Most Restrictive Target**

**Step 2: Determining the Average Grade to Most Restrictive Target (MRT)**

Figure 3-3 illustrates the concept of determining the average grade to the Most Restrictive Target (MRT). The local target table has the average grade over various groups of track fragments between the targets, as shown by the thin dotted line in Figure 3-3. This information allows the Braking Function to calculate the average grade from the trailing end of the train at its current worst case location to the MRT (Target T3 as in the above example), by taking a rolling average of the grades between the targets. This calculated average grade to MRT (shown by the thick dotted line in Figure 3-3) is then used for determining the profile velocity to the MRT.
Figure 3-3. Determining the Average Grade to Most Restrictive Target (MRT)

Step 3: Braking Profile Generation

The OBC enforces safe braking to the Most Restrictive Target (MRT) ahead of the train by generating a braking profile using the Braking Profile Algorithm (BPA). The BPA periodically (nominally once a second) calculates the maximum allowed (profile) velocity at the current worst-case location of the leading end of the train as it approaches the MRT. The mathematical model of the BPA is based upon the fundamental differential equations of kinematics, and is divided into three braking regions:

1. the “free run segment” which occurs during the brake propagation delay portion (i.e., from the time the brake application is requested by the OBC until the time that the actual braking effort begins);
2. the “transient braking segment” that occurs during the brake build up (i.e., from the time that the actual braking effort begins until the steady-state braking effort is achieved); and
3. the “steady state braking segment” which maintains a full constant braking effort until the target speed is achieved.

The general philosophy of the OBC braking algorithm is to periodically calculate (during each software cycle, not to exceed one second) the maximum speed that the train can be traveling at its current distance from the target location (over a specific grade scenario) and protect the train against exceeding that speed. This maximum speed will be referred to as the profile velocity and is defined as
the speed for which the associated worst-case full service braking distance (from that speed to the target speed) is equal to the leading end of the train’s worst-case distance to the target position. In order to calculate this speed, the following steps are necessary:

1. Determine the average grade between the current (worst-case) location of the trailing end of the train and the target location (see Figure 3-3); and

2. Determine the brake propagation delay time (Td), brake build up time (T), and the steady-state brake rate (α) as a function of the braking characteristics of the train consist.

The equations that describe the velocity and distance traveled for each of these braking regions and are used in the development of the profile velocity and time to penalty algorithms are defined in Figure 3-4.

\[ V_w = V_i + k g_{avg} T_d \]

\[ V_{ss} = \frac{V_i^2}{2} + \frac{k g_{avg} T^2}{2} \]

\[ s_{dT} = \frac{V_i T_d}{2} + \frac{k g_{avg} T_d^2}{6} \]

\[ s_{ss} = \frac{V_{ss}^2}{2} + \frac{k g_{avg} T^2}{2} \]

\[ s_{DTT} = \frac{V_i T_d}{2} + \frac{k g_{avg} T_d^2}{6} \]

where,

\[ \alpha \] is the steady-state brake rate (a.k.a. brake capacity) (mph/s)
sgeneric is the average grade of the track between the train’s worst-case trailing end location and the most restrictive target location (% grade)
k is the conversion factor needed to determine the effect of gravity on the stopping distance and is equal to 0.219 mph/s per %grade
SDTT is the trains distance to target (feet)
Sp is the distance that the train travels during the propagation delay (feet)
Sss is the distance that the train travels during constant braking (feet)
St is the distance that the train travels during the transient brake build up (feet)
T is the brake build up time (seconds)
Td is the brake propagation delay time (seconds)
Vf is the train velocity after the brake propagation delay (mph)
Vp is the profile velocity (mph)
Vss is the train velocity after the transient brake build up (mph)
VT is the target velocity (mph)

**Figure 3-4. Braking Regions and Profile Velocity vs. Distance Curve**

Using the above braking curve and definitions, the key operational equations (Equations 1 through 7 below) were derived for the Braking Profile Algorithm and are applied per the flowchart illustrated in Figure 3-5.

**Equation 1:** solve for distance \( S \) required to achieve the target speed at the instant that transient braking is complete:

\[
S = \left( v_p + \left( \frac{kg_{avg}}{2} + \frac{\alpha}{2} \right) T + \frac{kg_{avg} T^2}{2} \right) T_d - \left( \frac{1}{2} \left( \frac{kg_{avg}}{2} + \frac{\alpha}{6} \right) T^2 \right)
\]

**Equation 2:** solve for profile velocity \( v_p \):

\[
v_p^2 + (2\alpha T_d + \alpha T)v_p \left[ \frac{\alpha}{12} + \frac{4kg_{avg}}{T^2} + kg_{avg} \alpha T_d (T + T_d) + v_f^2 + 2(\alpha + kg_{avg}) S_{DTT} \right] = 0
\]

**Equation 3:** solve for distance \( S \) required to achieve the target speed at the instant that the brake propagation delay is complete

\[
S = v_f T_d + \frac{1}{2} kg_{avg} T_d^2
\]

**Equation 4:** solve for time \( t \)

\[
\left( \frac{\alpha}{6 T} \right) t^3 + \left( \frac{\alpha}{2 T} T_d + \frac{1}{2} kg_{avg} \right)t^2 \left( v_f + \frac{kg_{avg} T_d}{2} \right) t + \left[ S_{DTT} \left( v_f T_d + \frac{1}{2} kg_{avg} T_d^2 \right) \right] = 0
\]

**Equation 5:** solve for profile velocity \( v_p \) using the solution of \( t \):

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\[ v_p = v_T + k g_{avg} T_d + k g_{avg} t + \frac{\alpha}{2T} t^2 \]

**Equation 6:** solve for \( t_d \)

\[ \left( \frac{1}{2} k g_{avg} \right) t_d^2 + (v_T) t_d + S_{DTT} = 0 \]

**Equation 7:** solve for profile velocity \( v_p \) using the solution of \( t_d \):

\[ v_p = v_T + k g_{avg} t_d \]
1. Obtain $T_d$, $T$ and $\alpha$ values.
2. Calculate average grade from trailing end of the train to the selected target location.

Calculate distance $S$ required to reach the selected Target Speed Limit at the instant brake build-up is complete (i.e., within a time of $T_d+T$) using Equation 1.

- **Yes**
  - Steady-State braking at rate $\alpha$ is necessary to achieve target speed.
  - Calculate $V_p$ using Equation 2.

- **No**
  - Calculate distance $S$ required to reach target speed at the instant brake propagation delay is complete (i.e., within $T_d$), using Equation 3.

  - **Yes**
    - Is $S_{DTT} > S$?
      - Braking has to progress through $T_d$ and target speed will be achieved at sometime $t$ during brake build-up interval $T$.
      - Calculate time $t$ needed to achieve target speed, using Equation 4.
      - Calculate profile velocity $V_p$ using the solution for $t$ using Equation 5.

  - **No**
    - Target speed will be achieved at some time $t_d \leq T_d$. Solve Equation 6 to find $t_d$.
    - Calculate profile velocity $V_p$ using Equation 7.

Return to Braking Function

**Figure 3-5. Braking Profile Algorithm Flowchart**
Steps 4-5: Determine Enforceable Speed Limit (ESL) and Time to Penalty (TTP)

The profile velocity is calculated each software cycle for the worst case location of the leading end of the train, and in the event that the profile speed is greater than the current civil or temporary speed limit, the speed enforced is that of the restricting speed limit. Likewise, as the train approaches the target and the profile velocity becomes more restricting than the speed limit, the speed enforced is the profile velocity. In the case where the train’s actual speed is less than the calculated profile velocity, the Time To Penalty (TTP) is determined using the following expressions:

\[
\text{Distance to Penalty} = \frac{(v_p - v_a) \Delta S}{v_p - v_T} \text{ feet}
\]

\[
\text{Time To Penalty} = \frac{\text{Distance to Penalty}}{1.467 v_a} \text{ seconds}
\]

where \(v_a\) is the train’s current speed in miles/hour, \(v_p\) is the current profile velocity in miles/hour, \(v_T\) is the target speed in miles/hour, and \(\Delta S\) is the train’s actual distance to the target from its current position in feet.

TTP is defined as the time that the train may continue at its current speed until it intersects the braking profile curve and a penalty brake application is required. When the TTP is zero (i.e., the train’s actual speed is equal to the calculated profile velocity) a penalty brake application (equivalent to a Full Service Brake application) is initiated and maintained until the train reaches a stop.

3.3 Braking Function Warning and Enforcement

When the calculated Time To Penalty (TTP) is less than or equal to 40 seconds, the TTP is displayed to the train crew and an audible alert is given. Train movement at the current train speed will result in a continuous reduction in the TTP value. When the TTP reaches zero, the Braking Function applies the penalty brake for violation of the profile velocity. To avoid the penalty brake application, the operator

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needs to maintain a TTP > 0 seconds, and decelerate the train to make sure the speed does not break through the profile.

The Braking Function simply performs overspeed protection at its buffered Enforceable Speed Limit (ESL) when no other speed restrictions are present within 3 miles ahead of its current location. If the current velocity of the train is above the ESL, but under the buffered ESL, a warning indication will be displayed on the LDU. If the train velocity goes above the buffered ESL, then a penalty brake will be applied until the velocity is under the ESL.

4. **BRAKING PARAMETERS USED IN BPA**

Step 3 of the process of performing the braking function described the calculation the profile velocity using the Braking Profile Algorithm (BPA). As described in that step, the Braking Parameters [Td, T, α] must be calculated prior to the execution of the profile velocity equations. The values of [Td, T, α] are wholly dependent upon the braking characteristics of the train’s car and locomotive consist.

The expressions for the Braking Parameters for different train types can be derived by using the braking curves (speed vs. time plots) for a large number of consist configurations of each train type. For instance, the braking curve shown in Figure 4-1 is for an actual freight train. While the train was subjected to a full service brake application from an initial velocity to a stop on level track, the speed and distance traveled were recorded as a function of time. From these field data recordings, the braking curve as well as the rate of change of the train speed with time (dv/dt or deceleration) were calculated and plotted as shown in Figures 4-1 and 4-2.
As shown in Figure 4-2, a plot of the train deceleration as a function of time shows that the deceleration can be approximated by three straight line segments: a segment of zero deceleration during initial brake propagation, a segment of linearly increasing deceleration from zero to a constant rate during brake buildup, and a segment of deceleration at a constant rate until the train stops. These three segments are represented by the three Braking Parameters Td, T, and \( \alpha \):

\[
\begin{align*}
Td & = \text{the brake propagation delay time (seconds)} \\
T & = \text{the brake build up time (seconds)} \\
\alpha & = \text{the steady-state brake rate (a.k.a. brake capacity) (mph/s)}
\end{align*}
\]
One set of values for $T_d$, $T$, and $\alpha$ can thus be obtained from a single train run of a given consist. But, as the consists of a given train type vary in length, weight, mix of cars with different braking characteristics, etc., we must derive expressions for $T_d$, $T$, and $\alpha$ in terms of known consist parameters that can be determined accurately at train initialization. To be able to do this, we’ll need many braking curves representing various consists and their associated braking characteristics, from which different sets of $T_d$, $T$, and $\alpha$ values can be derived.

With sufficient sets of $[T_d, T, \alpha]$ values representing the breadth of consist characteristics for a given train type, we can derive expressions for $[T_d, T, \alpha]$ for that train type which require as input a minimum amount of consist information in addition to the fundamental values of train length and gross weight. The objective is to derive a set of expressions from which $[T_d, T, \alpha]$ can be calculated so that the Braking Function can predict the train stopping distance with sufficient accuracy, given any consist at any location on the track of the railroad.

Since it is impractical to obtain large numbers of field runs for different train types used by a railroad, an alternative approach can be used. This approach generates simulated train braking curves using models such as the TTCI’s TOES™ model. The TOES™ model is a computer program that models the longitudinal train dynamics, such as run-in and run-out events occurring from train handling. It includes non-linear models of draft gears, end-of-car cushioning devices, and a computational fluid dynamics model of the air brake system. It uses various train consist parameters such as the weight of each car and locomotive in a consist, the brake pipe length and pressure, type and number of brake valves on each car, the brake cylinder piston stroke, etc. The TOES™ model is thus completely different and independent from the kinematic equations model used in the braking function described in this paper.
4.1 Braking Parameter Expressions for Operational Efficiency

One of the goals of the authors was to derive the Braking Parameter Expressions (BPEs) that would in turn provide a high operational efficiency of the trains. For this purpose, operational efficiency is measured in terms of the overall run-time of a given train type over a given section of a railroad with known characteristics (grade, civil speed restrictions, bulletin-related speed restrictions, stops/starts, etc.), while always providing safe braking performance.

To determine the extent of input data required for the BPEs that would in turn provide acceptable operational efficiency of the trains, two sets of BPEs were derived for different train types:

1. “Simple” BPEs:
   - Td = f₁(Consist Length)
   - T = f₂(Consist Length)
   - α = f₃(Consist Weight per Brake Valve)

2. “Refined” BPEs:
   - Td = f₁(Consist Length, Weight per Brake Valve, # of Each Brake Valve Type, …)
   - T = f₂(Consist Length, Weight per Brake Valve, # of Each Brake Valve Type, …)
   - α = f₃(Consist Weight per Brake Valve, # of Each Brake Valve Type, …)

Braking parameters computed from the above BPEs and then used in actual train run simulations over a known railroad segment provided a certain level of braking performance measured in terms of run-time for a given train run.

Braking parameters computed from the Refined BPEs and then used in the same train run simulations as with Simple BPEs over a known segment of a railroad provided a slightly improved level of braking performance measured in terms of run-time for a given train run (less than 10% reduction in run time).

4.2 Braking Parameter Expressions Derivation

The key steps in deriving the BPEs applicable to trains that run on a railroad are shown below:
1. Survey the railroad’s fleet of trains and operation to determine train types and their key characteristics.

2. For each train type, use the selected model (such as the TOES™ Model) to determine the braking performance of an accurate and representative spectrum of possible consists that are typically run on the railroad. For each simulation, use the nominal set of consist parameter values that would be expected in real-world nominal operation. A general list of consist parameters that need to be considered in the model for different train types are listed in Table 4-1, along with their best case, nominal and worst case values.

3. For each train type, use the model simulation data in an iterative process and numerical methods (Multiple Linear or Non-Linear Regression that yields minimum sum of the squares of residual error) to derive a set of nominal BPEs (i.e. expressions for which the values of all the consist parameters are at their nominal values shown in Table 4-1) for Td, T, and α based on key dependent variables (e.g. Consist Length, Total Weight, Number of Brake Valves).

4. The nominal braking parameters computed from the simulations that use the nominal values of the consist parameters do not account for random variations in these parameters. Therefore, additional simulations are performed in which the car parameters are set at their extreme limits (best case and worst case values shown in Table 4-1) one at a time and the corresponding stopping distances are obtained. Probability density functions such as Beta distributions are then used to represent the variation of the stopping distance when the car parameters vary between their extreme limits.

5. Determine the effect of simultaneous consist parameter variations on the stopping distance via a convolution of the statistical distributions obtained in Step 4 and apply a correction factor to the nominal BPEs to account for the identified effects.
6. Determine the final set of adjusted BPEs considering all of the above factors.

From the above steps, the “Simple” BPEs for a given train type will turn out to be:

\[ s \]

\[ s \]

\[ \text{mph/s}. \]

In the above BPEs, \( a \) through \( f \) are constants and \( CF \) is a correction factor obtained from the convolution of the statistical distributions that represent the consist parameter variations.

Table 4-2 shows the nominal braking parameter expressions and the corresponding correction factors for different types of trains used on a U.S. railroad.

<p>| Table 4-1: Consist Parameters and Their Best Case, Nominal and Worst Case Values |
|---------------------------------|--------------|----------------|---------------------|-----------------------|</p>
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best Case Value</th>
<th>Nominal Value</th>
<th>Worst Case Value</th>
<th>Notes and Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locomotive Location and Distributed Power</td>
<td>Half at HE, Half 2/3 back, with DP</td>
<td>All HE, No DP</td>
<td>All HE, No DP</td>
<td>If the location of Distributed Power cannot be vitally determined, its benefits should be ignored in the Braking design.</td>
</tr>
<tr>
<td>Empty/Load Sensors</td>
<td>0% E/L equipped</td>
<td>7% E/L equipped</td>
<td>100% E/L equipped</td>
<td>Nominal E/L sensor percentages should be based on mix of population percentages for each individual car type. Also note that empty trains equipped with E/L sensors will take longer to stop, so “worst case” is actually MORE E/L sensors.</td>
</tr>
<tr>
<td>Percent Operable Brakes</td>
<td>100%</td>
<td>95%</td>
<td>85%</td>
<td>Per operating rule, a train leaving a terminal must have 100% operative brakes and at no time en route can the train operate with less than 85% operative brakes.</td>
</tr>
<tr>
<td>Load/Empty Count</td>
<td>All empties</td>
<td>+2% loaded</td>
<td>+10% loaded</td>
<td>Procedurally, the count should be very accurate, but assume a small chance of error resulting in extra unexpectedly loaded cars in an empty consist.</td>
</tr>
<tr>
<td>Vehicle Brake Force/Brake Shoe (given a 30-psi brake pipe reduction from a 90-psi brake pipe pressure)</td>
<td>13% Loaded NBR</td>
<td>10% Loaded NBR</td>
<td>8.5% Loaded NBR</td>
<td>Based on the range of allowable loaded Net Braking Ratios (NBR) from AAR Standard S-401, &quot;Brake Design Requirements&quot;. S-401 allows for loaded NBR in the range of 8.5% - 13% with a 30-psi brake pipe reduction from a 90-psi brake pipe pressure. Assume with good maintenance practices that this should fall around nominal value, but with some wider spread possible.</td>
</tr>
<tr>
<td>Parameter</td>
<td>Best Case Value</td>
<td>Nominal Value</td>
<td>Worst Case Value</td>
<td>Notes and Assumptions</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>-----------------</td>
<td>---------------</td>
<td>------------------</td>
<td>--------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Vehicle Brake Cylinder Piston Stroke</td>
<td>6 Inches</td>
<td>7.5 Inches</td>
<td>9 Inches</td>
<td>Range values from AAR data. Assume with good maintenance practices that this should fall around nominal value, but with some wider spread possible.</td>
</tr>
<tr>
<td>Nominal Brake Pipe Pressure</td>
<td>90 psi</td>
<td>88 psi</td>
<td>80 psi</td>
<td>Assume with good maintenance practices that the nominal pressure should fall around the best case value, but with some wider spread possible.</td>
</tr>
<tr>
<td>Vehicle Load</td>
<td>-10%</td>
<td>0%</td>
<td>+10%</td>
<td>Assume expected weight variation can vary from -10% to +10</td>
</tr>
<tr>
<td>Vehicle Brake Valve Type</td>
<td>All ABDX</td>
<td>39% ABDX</td>
<td>All ABD</td>
<td>Nominal brake valve type percentages should be based on mix of population percentages for each individual car type:</td>
</tr>
<tr>
<td>Ambient Temperature</td>
<td>100 deg F</td>
<td>20 deg F</td>
<td>-60 deg F</td>
<td>Average annual low (from National Weather Service) taken over territory is 22.4 deg F, round to 20 deg F which is also the mid-point of the range.</td>
</tr>
<tr>
<td>Vehicle Brake Pipe Length</td>
<td>-10%</td>
<td>Actual</td>
<td>+10%</td>
<td>Variation is expected between actual length and length available to OBC, although large variations are not expected. Assume very closely spread around nominal value.</td>
</tr>
<tr>
<td>% Grade Error</td>
<td>+0.1% grade</td>
<td>As stated grade</td>
<td>-0.1% grade</td>
<td>Concept: Models potential error in OBC grade database. Assume very closely spread around nominal value.</td>
</tr>
<tr>
<td>Train Speed Error</td>
<td>-0.5 mph</td>
<td>0 mph</td>
<td>+0.5 mph</td>
<td>Concept: Models potential train speed error in tachometers or OBC speed processing. Assume very closely spread around nominal value.</td>
</tr>
<tr>
<td>Vehicle Count</td>
<td>-5% cars</td>
<td>Correct Count</td>
<td>+5% cars</td>
<td>Concept: varies train weight/length by adding/subtracting loaded cars to/from the consist, assuming car counts can be off slightly. Procedurally the count should be very accurate, but assume a small chance of error.</td>
</tr>
<tr>
<td>Locomotive Weight</td>
<td>-5%</td>
<td>Actual</td>
<td>+ 5</td>
<td>Variation could be expected based on fuel weight, otherwise minimal. Assume very closely spread around nominal value.</td>
</tr>
<tr>
<td>Vehicle Length</td>
<td>-5%</td>
<td>Actual</td>
<td>+5%</td>
<td>Some variation is expected between actual length and length available to OBC, although large errors are not expected. Assume very closely spread around nominal value.</td>
</tr>
<tr>
<td>Ambient Pressure</td>
<td>14.0 psi</td>
<td>14.7 psi</td>
<td>14.7 psi</td>
<td>Assume very closely spread around nominal value.</td>
</tr>
<tr>
<td>Brake Pipe Leakage</td>
<td>5 psig/min</td>
<td>0.1 psig/min</td>
<td>0.1 psig/min</td>
<td>Leakage affects two items: propagation of brake signal early in the brake application and equalization pressure which can have conflicting effects on stopping distance. Therefore the effect of leakage varies by consist but typically, more leakage = faster braking so highest</td>
</tr>
<tr>
<td>Parameter</td>
<td>Best Case Value</td>
<td>Nominal Value</td>
<td>Worst Case Value</td>
<td>Notes and Assumptions</td>
</tr>
<tr>
<td>--------------------</td>
<td>----------------</td>
<td>---------------</td>
<td>------------------</td>
<td>--------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>leakage corresponds to best case value. Assume very closely spread around nominal value with good maintenance practices.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Can be ignored**: Vehicle Aerodynamic Resistance, Vehicle Brake Rigging Type, Vehicle Orientation, Control Relay Latency

---

### Table 4-2: Nominal “Simple” BPEs and Correction Factors for Different Train Types

<table>
<thead>
<tr>
<th>Train Type</th>
<th>Nominal Braking Parameter Expressions</th>
<th>Correction Factor (CF)</th>
</tr>
</thead>
</table>
| **Unit Tank Car Train** (Consists made up of ≥ 90% Tank Cars) | $T_d = 0.0007 \times (\text{Train Length}) + 3.01 \text{ s}$  
$T = 0.0092 \times (\text{Train Length}) + 5.0345 \text{ s}$  
$\alpha = 63.2558 \times (\text{Weight/Valve})^{0.9797} \text{ mph/s}$ | 1.26 |
| **Mixed Freight Train** (Consists made up of a mix of TOFCs, Tank Cars, Air Dumps, Box Cars, Gondola Cars) | $T_d = 0.0016 \times (\text{Train Length}) + 1.1135 \text{ s}$  
$T = 0.0013 \times (\text{Train Length}) + 28.7772 \text{ s}$  
$\alpha = 81.2417 \times (\text{Weight/Valve})^{-1.009} \text{ mph/s}$ | 1.23 |
| **Intermodal Train** (Consists made up of ≥ 90% Flat Cars) | $T_d = 0.0023 \times (\text{Train Length}) + 1.828 \text{ s}$  
$T = 0.0186 \times (\text{Train Length}) + 0.3265 \text{ s}$  
$\alpha = 82.5578 \times (\text{Weight/Valve})^{-0.9769} \text{ mph/s}$ | 1.20 |
| **Unit Steel Hoppers Train** (Consists made up of mostly Steel Hoppers) | $T_D = 0.000936 \times (\text{Train Length}) + 3.0896 \text{ s}$  
$T = 0.0087 \times (\text{Train Length}) + 5.7639 \text{ s}$  
$\alpha = 70.9318 \times (\text{Weight/Valve})^{0.9819} \text{ mph/s}$ | 1.19 |
| **Unit Mixed Hoppers Train** (Consists made up of a mix of Steel and Aluminum Hoppers) | $T_D = 0.0007 \times (\text{Train Length}) + 3.1661 \text{ s}$  
$T = 0.009 \times (\text{Train Length}) + 4.6034 \text{ s}$  
$\alpha = 26.7060 \times (\text{Weight/Valve})^{-0.7508} \text{ mph/s}$ | 1.32 |
<table>
<thead>
<tr>
<th>Train Type</th>
<th>Nominal Braking Parameter Expressions</th>
<th>Correction Factor (CF)</th>
</tr>
</thead>
</table>
| Conventional Passenger Train (Consists made up of mostly Passenger Cars with the Locomotives at Head End only). | $T_D = 0.0013 \*(\text{Train Length}) + 1.8322 \text{s}$  
$T = 0.0128 \*(\text{Train Length}) + 5.2526 \text{s}$  
$\alpha = 34.0638 \*(\text{Weight}/\text{Valve})^{0.6546} \text{ mph/s}$ | 1.11 |
| Push-Pull Passenger Train (Consists made up of mostly Passenger Cars with the Locomotives in a push-pull configuration) | $T_D = 0.003\*(\text{Train Length}) + 1.1299 \text{s}$  
$T = 0.0111\*(\text{Train Length}) + 5.0513 \text{s}$  
$\alpha = 53.6164 \*(\text{Weight}/\text{Valve})^{0.7255} \text{ mph/s}$ | 1.08 |
| Mixed Passenger/Freight Train (Consists made up of a mix of Passenger Cars and Freight Cars) | $T_D = 0.0012\*(\text{Train Length}) + 2.3556 \text{s}$  
$T = 0.0158\*(\text{Train Length}) + 0.2727 \text{s}$  
$\alpha = 5.8264 \*(\text{Weight}/\text{Valve})^{0.4438} \text{ mph/s}$ | 1.19 |

### 4.3 Input Data for BPEs

Typically, the functional group of the railroad that is charged with putting together train consists of different train types for revenue service deployment sends consist data to the CAD Subsystem of their PTC System. The data include both gross data such as consist length, consist weight, engine IDs and positions, and the number of loaded and empty cars in the consist; as well as a detailed train list showing the precise sequence, types (kinds), and weights of each engine and car in the consist.

The CAD Subsystem can then send the required input data to the On-Board Computer (as shown in Figure 2), which in turn uses the data in the BPEs to compute the braking parameters for use in the BPA. If the railroad considers the operational efficiency obtained with the Simple BPEs as satisfactory, only three pieces of input data (viz., consist length, gross weight, and number of brake valves) are needed to generate the BPEs shown in Table 4-2. Note that most freight cars (with the exception of long Flat Cars with Drawbar Face Length ≥ 165 ft) and passenger cars have one brake valve each. Hence the number
of brake valves is considered equal to the number of cars in the train consist. Sometimes, a small but unknown number of long Flat Cars (that have two brake valves each) may exist in a given freight train consist. For such consists, assuming the number of brake valves is equal to the number of cars gives conservative results.

The input data must be validated by the train crew and the dispatcher initially at the time of putting the train consist in revenue service, and every time the consist is changed, for example, during set-out and pick-up (SOPU) operations.

5. VALIDATION OF BRAKING FUNCTION

The braking function can be validated both analytically and with the help of field testing on a railroad. The analytical approach consists of three steps:

1. Selecting a statistically significant number of consists of different train types that were actually run on a railroad, computing their BPEs, and obtaining the stopping distances for them under different grade scenarios by using the BPEs in a laboratory model of the BPA.

2. Using an independent model such as the TTCI TOES™ Model to run the same consists under the same grade scenarios to obtain a distribution of stopping distances for randomly selected consist parameter values in a Monte Carlo simulation approach.

3. Comparing the stopping distances obtained from the above two steps.

The braking function can also be validated with the help of field testing. A statistically significant set of braking scenarios with selected train consists over selected track segments of a railroad can be run with the braking function software installed in the OBCs of controlling locomotives of the train consists. The runs should be made at different speeds up to the MAS limits governing the selected track segments with different grade characteristics (flat, large positive and negative grades, transitions between positive...
and negative grades, etc.). No revenue passenger trains or freight trains with hazardous materials should be used in these tests.

6. SUMMARY

This paper presented a braking function that can be implemented in the On-Board Computer of a Positive Train Control System. The braking function enforces safe braking to the Most Restrictive Target (MRT) ahead of the train by generating a braking profile using the Braking Profile Algorithm (BPA). The mathematical model of the BPA is based upon the fundamental differential equations of kinematics, and is divided into three braking regions:

6. the “free run segment” which occurs during the brake propagation delay portion (i.e., from the time the brake application is requested by the OBC until the time that the actual braking effort begins), represented by a Braking Parameter called brake propagation delay time $T_d$ (seconds);

7. the “transient braking segment” that occurs during the brake build up (i.e., from the time that the actual braking effort begins until the steady-state braking effort is achieved) represented by a Braking Parameter called brake build up time $T$ (seconds); and

8. the “steady state braking segment” which maintains a full constant braking effort until the target speed is achieved, represented by a Braking Parameter called steady-state brake rate $\alpha$ (mph/s).

Braking Parameter Expressions (BPEs) are derived in terms of known train data (namely, consist length, gross weight, and the number of cars). A key feature of the BPE derivation process is the use of a detailed sensitivity analysis and a novel convolution process to account for real-world variations in train consist parameters in a reasonable and practical way, rather than assuming worst case values for all parameters which would result in unacceptably restrictive braking performance. This process gives a
correction factor to be applied on the nominal BPEs in order to ensure an acceptably low probability (e.g. 0.00001) of exceeding a target stopping point if multiple consist parameters tend towards their worst-case values simultaneously.

7. ACKNOWLEDGEMENTS

The work presented in this paper was done partly on an Ansaldo STS contract with a U.S. Regional Railroad. Funding for the contract was partly provided by the Federal Railroad Administration. Technical contributions to this paper from the Regional Railroad, Rail Safety Consulting LLC, the Federal Railroad Administration, and the Transportation Technology Center, Inc. are gratefully acknowledged.
A Simple and Efficient Train Braking Algorithm for PTC Systems

Chinnarao Mokkapati
Robert D. Pascoe
Ansaldo STS USA
Outline

- Introduction
- Basic requirements to be met by Braking Function
- Braking Function Implementation
  - Braking Profile Algorithm
- Braking Parameter Expressions
- Accounting for Consist Parameter Variations
- Braking Function Validation
Introduction

How do you control the speed of a 100-car freight train hauling 10,000 tons of load safely and efficiently while obeying all speed restrictions encountered along the way, including stopping at a zero speed target?

- Many models and methods are available
- Make a lot of conservative assumptions
- Not very efficient

This paper presents a method that we believe is both simple and efficient, while always ensuring safety.

- Safe speed enforcement
- Penalty braking
Braking Function Requirements: Train Types

A Train Braking Function must ensure safe braking for all consist variations of different train types with different power arrangements:

- Freight trains, with a mix of different types of freight cars
- Unit hopper trains (all steel, all aluminum or mix of hoppers)
- Unit tank car trains
- Intermodal trains
- Passenger car trains
- Mixed passenger car/freight car trains
- Etc.
Braking Function Requirements: Enforcement

The Braking Function shall ensure and enforce safe train speed, on (or approaching) mapped track, in both forward and reverse directions, for all of the following conditions:

a. approaching authority limits;
b. approaching Form A, B and S bulletin limits;
c. approaching civil speed restrictions;
d. approaching switches requiring position confirmation;
e. approaching Form C restrictions;
f. approaching signals in CTC;
g. approaching controlled track from uncontrolled track;
h. maintaining speed at or below the relevant speed limit on controlled, mapped/uncontrolled and unmapped/uncontrolled track;
i. approaching OS and Track Integrity circuits in DTC indicating OCCUPIED.
Braking Function Requirements: Penalty Braking

The Braking Function shall enforce penalty braking of a train which violates any of the following conditions:

a. when movement is required at restricted speed (e.g., 20R/10R or any speed while in Restricted Speed mode
b. buffered civil speed limits;
c. buffered restricted speeds within Form A limits;
d. active Form S buffered speed restrictions;
e. Form C buffered speed restrictions;
f. speed limit defined by signal indication at stop or “restricting”;
g. Form B limits without correct PIN entry;
h. movement authority limits;
i. signals at stop without permission to pass;
j. entering controlled track from uncontrolled track without authority;
k. traversing a switch requiring position confirmation without confirmation;
l. traversing a switch with out-of-correspondence or incorrect position for route;
m. Track Integrity circuit or OS circuit indicating OCCUPIED in DTC at higher than 20 mph restricted speed.
Braking Function Implementation

- Braking Function is implemented in a vital On-Board Computer (OBC), which also performs a Location Determination Function (LDF).
- OBC has a database from which a Local Target Table (sliding window) of targets within 3 miles of train’s current location is created.
- Braking Function needs the train info:
  - Current location and speed (from the LDF)
  - Current speed limit, target locations, target speed limits, average grade between targets (from the database)
Braking Function: Five Step Process

Using target data from Local Target Table, the Braking Function:

1. Determines the Most Restrictive Target (MRT) ahead of the train’s current position considering the associated speed restrictions and distances.
2. Calculates the average grade between the train’s trailing end location (worst case) and the selected target.
3. Using the Braking Profile Algorithm, calculates the Profile Velocity to the selected target.
4. Sets the Enforceable Speed Limit (ESL) as the lower of a) the MAS applicable to the train’s current location or b) the profile velocity that the train must adhere to in order to avoid a penalty brake application as the train approaches a speed restriction associated with the selected target.
5. Calculates the Time to Penalty (TTP) associated with the selected target.
Overall Braking Function Flowchart

See Figure 3-1 in the Paper
Consider the following four targets T1, T2, T3, and T4:

- MRT is selected as Target T3, so Profile P3 is followed.
Braking Function calculates the average grade from the trailing end of the train at its current worst case location to the MRT (Target T3 as in the above example), by taking a rolling average of the grades between the targets.

The calculated average grade to MRT is then used for determining the profile velocity to the MRT.
Step 3: Braking Profile Generation

The mathematical model of the BPA is based upon the fundamental differential equations of kinematics, and is divided into three braking regions:

1. the “free run segment” which occurs during the brake propagation delay portion (i.e., from the time the brake application is requested by the OBC until the time that the actual braking effort begins);
2. the “transient braking segment” that occurs during the brake build up (i.e., from the time that the actual braking effort begins until the steady-state braking effort is achieved); and
3. the “steady state braking segment” which maintains a full constant braking effort until the target speed is achieved.
Braking Regions and Profile Velocity vs. Distance

\[
V_{f} = V_{p} - \frac{kg_{avg}}{2}T_d = \text{train velocity after brake propagation delay (dependent on grade)}
\]

\[
V_{ss} = V_{p} - \left[ \frac{kg_{avg}}{2}T_d \right] = \frac{kg_{avg}}{2}T_d - \left( \frac{kg_{avg}}{2} + \frac{a}{2} \right)T
\]

\[
S_{p} = V_{p}T_d - \frac{1}{2}kg_{avg}T_d^2
\]

\[
S_{t} = V_{t}T - \left( \frac{1}{2}kg_{avg} + \frac{a}{6} \right)T^2
\]

\[
S_{ss} = \frac{(V_{p} - kg_{avg}T_d)T}{2(a + kg_{avg})} - \left( \frac{kg_{avg}}{2} + \frac{a}{2} \right)T^2
\]

\[
S_{D} = S_{p} + S_{t} + S_{ss}
\]
Braking Profile Algorithm

Braking Profile Algorithm periodically calculates the profile velocity that the train can be traveling at its current distance from the target location and protects the train against exceeding that speed.

The profile velocity calculation needs two pieces of data:

1. Average grade between the current (worst-case) location of the trailing end of the train and the target; and
2. Braking Parameters for the given train as a function of the braking characteristics of the train consist.
   - Brake propagation delay time (Td)
   - Brake build-up time (T)
   - Steady-state brake rate (\(\alpha\))
Braking Profile Algorithm Flowchart

See Figure 3-5 in the Paper
Steps 4 and 5: Determine ESL and TTP

- If the profile speed is greater than the current speed limit (MAS), the speed enforced (ESL) is the current speed limit.
- Likewise, as the train approaches the target and the profile velocity becomes more restricting than the speed limit, the speed enforced is the profile velocity.
- In the case where the train’s actual speed is less than the calculated profile velocity, the Time To Penalty (TTP) is determined.
  - TTP is defined as the time that the train may continue at its current speed until it intersects the braking profile curve and a penalty brake application is required.
- When the TTP is zero, a penalty brake application (equivalent to a Full Service Brake application) is initiated and maintained until the train reaches a stop.
When the calculated Time To Penalty (TTP) is less than or equal to 40 seconds, the TTP is displayed to the train crew and an audible alert is given.

Train movement at the current train speed will result in a continuous reduction in the TTP value.

When the TTP reaches zero, the Braking Function applies the penalty brake for violation of the profile velocity.

To avoid the penalty brake application, the operator needs to maintain a TTP > 0 seconds, and decelerate the train to make sure the speed does not break through the profile.
The Braking Function simply performs overspeed protection at its buffered Enforceable Speed Limit (ESL) when no other speed restrictions are present within 3 miles ahead of its current location.

If the current velocity of the train is above the ESL, but under the buffered ESL, a warning indication will be displayed on the LDU.

If the train velocity goes above the buffered ESL, then a penalty brake will be applied until the velocity is under the ESL.
Braking Parameters Td, T and Alpha

How do we derive the expressions for the Braking Parameters?
Nominal Braking Parameter Expressions

TOES™ Simulation Data
- Time (sec)
- Distance (feet)
- Deceleration (mph/min)
- Velocity (mph)
- Brake Cylinder Pressure (psi)

Plot of all $T_d$ Data Points
Best-Fit Equation for $T_d$

Plot of all $T$ Data Points
Best-Fit Equation for $T$

Plot of all $\alpha$ Data Points
Best-Fit Equation for $\alpha$
Nominal Braking Parameter Expressions

Examples

- **Unit Steel Hopper Train**
  \[ T_D = 0.000936 \times \text{(Train Length)} + 3.0896 \text{ s} \]
  \[ T = 0.0087 \times \text{(Train Length)} + 5.7639 \text{ s} \]
  \[ \alpha = 70.9318 \times \left( \frac{\text{Weight}}{\text{Valve}} \right)^{-0.9819} \text{ mph/s} \]

- **Intermodal Train**
  \[ T_d = 0.0023 \times \text{(Train Length)} + 1.828 \text{ s} \]
  \[ T = 0.0186 \times \text{(Train Length)} + 0.3265 \text{ s} \]
  \[ \alpha = 82.5578 \times \left( \frac{\text{Weight}}{\text{Valve}} \right)^{-0.9769} \text{ mph/s} \]
Step 1: Determine the distribution shape and values for each parameter, using the nominal value as the mode and the best case and worst case values as the extreme boundaries.

Step 2: Convert the parameter distributions to the common scale of the change in stopping distance from nominal as the parameter in question varies per its distribution.

Step 3: Select the parameters to be convolved and perform the convolution to obtain a single distribution of stopping distances, then select the probability threshold to be applied to the convolved distribution to find the safe braking offset.

- This parameter reflects the probability of not exceeding a given stopping target, given the expected variations of the selected key parameters.
- For example, a threshold of P=0.99999 (99.999%) can be used.
Example for Percent Operable Brakes – range and shape determined by end points and desired “mode”. See Table 4-1.

Mode represents the point at which there is an equal 50% probability of the value being higher or lower than the mode value.
Convolution Step 2: Map to Stopping Distance Impact

Parameter Value

Parameter Impact on Stopping Distance from Nominal, feet

Selected Parameter Shape Distribution

Probability

New shifted best case point = 280 ft

Best case point from TOES

-400 -300 -200 -100 0 +100 +200 +300 +400 +500 +600 +700 +750 (worst)

95% (nominal)

95%

96%

94%

92%

90%

88%

86%

85% (worst)

LOCK

0 feet (nominal)
Convolution Step 3: Combine Distributions

Best Case Parameter Changes (decreased stopping distance) vs. Worst Case Parameter Changes (increased stopping distance)

-1500 to +1500 feet impact on stopping distance from nominal.

Probability of not exceeding this stopping distance = 99.999%
Examples

- **Mixed Freight Train**
  \[ T_d = 1.23 \times \{0.0016 \times (\text{Train Length}) + 1.1135\} \text{ s} \]
  \[ T = 1.23 \times \{0.0013 \times (\text{Train Length}) + 28.7772\} \text{ s} \]
  \[ \alpha = (1/1.23) \times 81.2417 \times (\text{Weight/Valve})^{-1.009} \text{ mph/s} \]
  Correction Factor = 1.23

- **Conventional Passenger Train**
  \[ T_D = 1.11 \times \{0.0013 \times (\text{Train Length}) + 1.8322\} \text{ s} \]
  \[ T = 1.11 \times \{0.0128 \times (\text{Train Length}) + 5.2526\} \text{ s} \]
  \[ \alpha = (1/1.11) \times 34.0638 \times (\text{Weight/Valve})^{-0.6546} \text{ mph/s} \]
  Correction Factor = 1.11
Safety Validation of Braking Function

- Consider a statistically significant number of consists of different train types and compute the OBC stopping distances for them under different grade scenarios.
- Run the same consists under the same grade scenarios, using the TOES™ model.
- The distribution of stopping distance results between the OBC and the TOES™ Model can be compared to validate the final Braking Parameters.
- To provide a high degree of confidence in the comparison process, the TOES™ model can be run with large suite of randomly selected consist parameter values for each consist in a Monte Carlo simulation approach.
Final Safety Validation of Overall Braking Function

Field tests should be used to demonstrate and verify that the analytical safety validation performed above is sufficient to establish a high level of confidence in the OBC Braking Function’s ability to safely stop a train of any type over any track segment.

To reach this objective, a statistically significant set of braking scenarios with selected train consists over selected topography, should be defined and tests should be run with the final Braking Function software installed in the OBCs of the controlling locomotives of the consists.
Suggested Braking Runs for Each Consist

- Brake application on near level track, forward and reverse.
- Brake application on positive to negative grade transition.
- Brake application on a varying positive grade.
- Brake application on a large constant positive grade.
- Brake application on a negative to positive grade transition.
- Brake application on a varying negative grade.
- Brake application on a large constant negative grade.
Questions?