Computer-Aided Analysis of Non-Coded Alternating-Current Track Circuits
Using a Finite-Element Transmission-Line Model

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ABSTRACT

Track circuits are conceptually simple but the electrical characteristics of railway track add complexity to their operation and understanding. To assist with analysis, a software tool is presented that models an alternating-current track circuit with a two-element track relay. The track is modeled as a transmission line, which is typical of alternating-current track circuit analysis, but instead of using the two-port parameter solution to the Telegrapher’s Equations, a finite number of transmission-line elements of finite length are analyzed. Simple network analysis of the resulting ladder topology involves only Ohm’s law and arithmetic instead of the hyperbolic trigonometry of the exact solution.

Whereas the Telegrapher’s Equations solution is limited to finding feed voltage and current in terms of relay conditions, this analysis calculates from feed to relay so that relay voltage, current and phases can be determined for varying feed voltages. Further, it allows examining the effect of a shunt placed anywhere along the track, something which cannot be easily predicted by equations that assume uniform distribution of primary transmission-line constants.

The user has control over feed voltage and frequency, feed- and relay-end reactors, track wire resistances, impedance bond values and impedances of track relay coils. Rail resistance and inductance, ballast resistance and capacitance, and track length may be varied. A shunt of any value may be placed at any location along the track and its effects observed. Numerical output includes both magnitude and phase angle for all complex quantities. The tool generates graphs of rail-to-rail voltage, rail current and ballast impedance as functions of distance; a vector diagram graphically shows the phase relationships of track- and local-coil currents and voltages in the track relay.

INTRODUCTION

Track circuits have been the foundation of signal systems for well over a century. They reliably detect the presence of trains or other adverse rail conditions in a fail-safe manner. This feature forms the basis for safe operation; for example, trains can establish their own protection to safely set signals for following trains without relying on human intervention [1].

Early track circuits used direct current. Alternating-current track circuits were introduced to avoid interference where direct-current propulsion used the rails for traction-power return. (The model here also works with direct current by setting the frequency to zero.) This made analysis more complex as phase angles must be considered in addition to magnitudes of operating characteristics.

Signal current leakage between rails and through the ballast occurs throughout the track circuit and causes the rail-to-rail voltage, rail current and leakage current to vary continuously from feed to relay. Early center-leak analysis approximated the leakage as a single lumped element at the center of the track circuit. It was a simple approximation and not always accurate [2].

In the early twentieth century, L. V. Lewis applied transmission-line theory to the analysis of track circuits [2]. Transmission-line theory was developed in the nineteenth century in connection with the study of telegraph lines [3]. The theory describes the behavior of alternating currents and voltages in these lines and also applies to other transmission media such as telephone lines and radio-frequency coaxial cables. The transmission line under consideration consists of two parallel conductors separated by a dielectric insulation. The transmission-line model
consists of many repeating elements, each consisting of: a series impedance representing the elemental resistance and inductance of the conductors; and a shunt admittance representing the elemental conductance and capacitance of the dielectric material between the conductors. As the number of elements increases, the model approaches the true distributed nature of the transmission medium characteristics. Using calculus, the number of elements is allowed to approach infinity while their lengths approach zero. The resulting pair of differential equations are called the \textit{Telegrapher’s Equations} and describe the voltage and current along the transmission line with perfectly distributed characteristics.

When considering railway track as the transmission medium from the feed to the relay, the conductor impedance is the rail resistance and inductance in series, and the dielectric admittance is the ballast conductance and capacitance in parallel. (Here, ballast resistance, $B$, is considered instead of ballast conductance, $G$.) Figure 1 shows the equivalent circuit for an element of the track; it is similar to the elements used in any transmission-line application. Now the rail impedance and rail-to-rail leakage can be uniformly distributed throughout the track circuit instead of lumped at the center, allowing a more accurate \textit{distributed-leak} analysis.

Lewis used the following solution to the Telegrapher’s Equations that gives feed-end voltage and current, $E$ and $I$, in terms of relay-end voltage and current, $e$ and $i$ [2] [3] [4] [5] [6] [8]:

\begin{align}
E &= e \cosh \sqrt{ZY} + i \frac{Z}{\sqrt{Y}} \sinh \sqrt{ZY} \\
I &= i \cosh \sqrt{ZY} + e \frac{Y}{\sqrt{Z}} \sinh \sqrt{ZY}
\end{align}

where $Z$ and $Y$ are the total equivalent rail impedance and ballast admittance, respectively, for the entire track circuit (found by multiplying impedance per length and admittance per length by length). The usual appearance of conductance, $G$, is replaced here with admittance, $Y$, to include ballast capacitance. These equations are simple enough to be evaluated with slide rule and trigonometric tables—the only tools available at the time they were first applied and for many years afterward.

Equations (1) and (2) are the \textit{two-port parameter solution} to the Telegrapher’s Equations since the track is treated as a two-port network, i.e. only the input and output voltages and currents are considered. As shown in Figure 2, the track itself is a black box; these equations do not expose the behavior along the track nor do they allow non-uniform modifications to the track characteristics. In other words, the individual elements of Figure 1 are not accessible.
In the model here, shown in Figure 3, a compromise is made between a single lumped element (center-leak method) and an infinite number of elements (distributed-leak method with transmission-line equations). With this quasi-distributed leak, there is a large, but finite, number of elements. Voltages and currents are calculated at each element. The number of elements is high enough to give results that closely agree with the exact transmission-line solution. The number of calculations involved makes it impractical for pencil-and-paper analysis, but this is easily overcome by taking advantage of today’s ubiquitous computing power.

Placing a test shunt across the rails disturbs the uniform distribution of transmission-line characteristics such that a single set of transmission-line equations are not valid for a complete shunted track circuit. Since the analysis presented here individually computes voltages and currents at each element throughout the track, uniform distribution of track parameters is no longer a requirement. A test shunt of any impedance can be placed at any track element and its effects calculated in the same manner as the un-shunted track.

MODEL

Figure 4 shows the circuit used to model the entire track circuit, including the feed- and relay-end components and the track (transmission-line) model. A test shunt can be placed in parallel with any of the ballast impedances; element 0 is a placeholder for a test shunt at the feed end of the track.
Nomenclature

Voltages, currents, impedances and power are complex numbers, i.e. they have both magnitude and phase angle. Feed- and relay-end parameters are represented by upper- and lower-case letters, respectively. Symbols for rail-to-rail currents, such as ballast leakage and bond current, are modified by a hat ('). Prime (') denotes with respect to distance.

\[ E_s \text{ feed transformer secondary voltage (V)} \]
\[ f \text{ source frequency (Hz)} \]
\[ \omega \text{ source radian frequency (rad/s)} \]
\[ I_s \text{ total current from feed transformer secondary (A)} \]
\[ S, P, Q \text{ total, real and reactive power delivered by feed transformer (VA, W, VAR)} \]
\[ Z_s \text{ total impedance seen by feed transformer secondary (Ω)} \]

Figure 4—Equivalent circuit of feed-end, track and relay-end components. Down-arrows (↓) indicate the mutually-exclusive locations of test shunts. Element 0 is a dummy element for applying a test shunt at the feed end.
\( Z_o, z_o \) feed-end, relay-end reactor impedances (Ω)
\( R_w, r_w \) feed-end, relay-end track wire resistances (Ω)
\( Z_b, z_b \) feed-end, relay-end bond impedances (Ω)
\( I_o, i_o \) feed-end, relay-end track currents (A)
\( E, e \) feed-end, relay-end rail-to-rail voltages (V)
\( I, i \) feed-end, relay-end track currents (A)
\( E, \bar{E} \) and \( I, \bar{I} \) as calculated from \( e \) and \( i \) using transmission-line equations (1) and (2) (V, A)
\( z \) impedance of relay end, including bond (Ω)
\( e_r, e_L \) relay track-coil, local-coil voltages (V)
\( i_r, i_L \) relay track-coil, local-coil currents (A)
\( Z_r, Z_L \) relay track-coil, local-coil impedances (Ω)

Track characteristics (rail resistance and inductance are for two rails):
\( R' \) rail resistance (Ω/kft)
\( L' \) rail inductance (H/kft)
\( B' \) ballast resistance (Ω·kft)*
\( C' \) ballast capacitance (F/kft)
\( Z_R \) rail impedance (Ω/kft)
\( Z_B \) elemental rail impedance (Ω)
\( Z \) total rail impedance (Ω)
\( Y \) total ballast admittance (S)
\( \ell \) track circuit length (ft)
\( N \) number of elements
\( \Delta x \) element length (ft)
\( E_i \) rail-to-rail voltage at output of \( i^{th} \) track element (V)
\( I_i \) rail current entering \( i^{th} \) track element (A)
\( I_i \) leakage current at \( i^{th} \) track element (A)
\( Z_i \) impedance looking into \( i^{th} \) track element (Ω)
\( Z_{sh} \) impedance of test shunt (Ω)

*Since ballast resistance is a \textit{shunt} resistance (not series) its units are ohms \textit{times} distance (not ohms per distance). For more elegant equations, one can instead use ballast \textit{conductance}, \( G' \), in units of S/kft (siemens per distance) so that distance is always in the denominator; this is used in much of the early literature [2] [5] [6]. As a practical matter, ballast \textit{resistance}, \( B' \), is used here since it is more usual to measure and think in ohms.

\textbf{Analysis}

To simplify many of the equations, define a parallel-impedance function:
\[
Z_1 \parallel Z_2 = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{Z_1 Z_2}{Z_1 + Z_2}
\] (3)

To find the elemental track characteristics, start with the track’s electrical characteristics with respect to length and adjust for the track circuit length and number of elements. The factors of 1000 are for conversion between feet and kilofeet.
\[
\Delta x = \frac{\ell}{N}
\] (4)
\[
R = R' \frac{\Delta x}{1000}, \quad L = L' \frac{\Delta x}{1000}, \quad B = B' \frac{1000}{\Delta x}, \quad C = C' \frac{\Delta x}{1000}
\] (5)

Then (see Figure 1),
\[
Z_R = R + j \omega L
\] (6)
\[
Z_B = B \| \frac{1}{j\omega C} = B \| \frac{-j}{\omega C} = \frac{B}{\omega + B^2C^2 + 1} - j\frac{\omega B^2C}{\omega^2B^2C^2 + 1}
\]

where \(\omega = 2\pi f\). Rail inductance and ballast capacitance are entered by the user in units of mH/kft and \(\mu\)F/kft, respectively, and must be divided by \(10^3\) and \(10^6\), respectively, before using in equation (5). If the rail impedance is available only as a magnitude and angle per thousand feet, \(Z' R \angle \theta\), convert it to resistance and inductance per thousand feet:

\[
R' = Z' R \cos \theta \quad \text{and} \quad L' = \frac{1}{2\pi f} Z' R \sin \theta.
\]

For example, if the rail impedance is given as 0.1 \(\Omega/kft\) \(\angle 60^\circ\), then \(R' = 0.05 \Omega/kft\) and \(L' = 0.551 \text{ mH/kft}\) at 25 Hz. If magnitude and power factor, \(p\), are given, then find the angle as \(\theta = \cos^{-1} p\) and proceed as above.

To find voltages and currents throughout the circuit, we need to know the total current from the feed transformer. To find the total current, we need to know the total impedance of the circuit, which is found by starting at the relay end and working back toward the feed. At the relay end and the last two track elements,

\[
z = z_b \| (r_w + z_x + z_r)
\]

For the \(i^{th}\) track element,

\[
Z_i = Z_R + (Z_B \| Z_{i+1})
\]

At the first two track elements and the feed end,

\[
Z_o = Z_1 = Z_R + (Z_B \| Z_2)
\]

Now find total current and all individual currents and voltages from the feed to the relay. The feed voltage, \(E_s\), is the phase reference for all internal calculations, so its phase angle is 0°; that is, \(E_s = |E_s|\angle0^\circ\). Note that equations (15), (20), (23), (26), (28) and (35) implement the current divider formula [7].

\[
I_s = \frac{E_s}{Z_s}
\]

\[
I_b = I_s \frac{Z_0}{Z_0 + Z_b}
\]

\[
E = E_0 = E_s - I_s(Z_x + R_w)
\]

\[
l = I_0 = I_1 = I_s - I_b
\]

\[
I_0 = 0
\]

\[
E_1 = E_0 - I_1Z_R
\]

\[
I_1 = I_1 \frac{Z_2}{Z_2 + Z_B}
\]

For the \(i^{th}\) track element,

\[
l_i = I_{i-1} - I_{i-1}
\]
\[ E_i = E_{i-1} - I_i Z_R \]  
\[ I_i = I_{i-1} \frac{Z_{i+1}}{Z_{i+1} + Z_B} \]  

At the last track element and the relay end,  
\[ I_N = I_{N-1} - \hat{I}_{N-1} \]  
\[ e = E_N = E_{N-1} - I_N Z_R \]  
\[ \hat{I}_N = I_N \frac{z}{z + Z_B} \]  
\[ i = I_N - \hat{I}_N \]  

The relay current, relay voltage and relay-end bond current are  
\[ i_r = \frac{I_r}{z_b + \frac{r_w}{z_b} + z_x + z_r} \]  
\[ e_r = i_r z_r \]  
\[ \hat{i}_b = i - i_r \]  

The track local voltage is fixed at 110 V ac; other values would affect the magnitude of the local current but not its phase. For the track local coil, which is fed from the same source as the feed and thus is in phase with the feed,  
\[ e_L = 110 \angle 0^\circ \]  
\[ i_L = \frac{e_L}{z_L} \]  

Finally, the total complex power required from the feed transformer is  
\[ S = E_s^* I_s^* \]  

where the asterisk (*) represents the complex conjugate. Real power, \( P \), and reactive power, \( Q \), are the real and imaginary components, respectively, of the complex power:  
\[ P = \Re(S) \quad Q = \Im(S) \]  

When a test shunt is applied from rail to rail, equations for the affected track element are modified to account for the test shunt impedance in parallel with the ballast impedance. For example, if a test shunt of impedance \( Z_{sh} \) is placed at element \( i \), equation (23) becomes  
\[ \hat{I}_i = I_i \frac{Z_{i+1}}{Z_{i+1} + (Z_B || Z_{sh})} \]  

**User Interface**

A screenshot of the analysis tool is shown in Figure 5. Across the top of the window are user-entry text boxes for feed-end, track and relay-end parameters. The feed voltage can also be swept using up- and down-arrow keys to facilitate finding the feed voltage for particular relay operating characteristics. Below the track parameters are controls for the test shunt: its impedance; a checkbox to apply and remove the shunt; and a slider to adjust its position along the track. Below the feed-end and relay-end parameters are calculated values. Unlike internal calculations, all displayed voltage, current and power phase angles are with respect to the track coil current of the track relay; that is, \( i_r = |i_r|\angle 0^\circ \).
Across the bottom of the window, calculations are presented in tabular and graphical form.

- In the lower-left is a text box with various calculated values, including the feed-end voltage and current as determined by the transmission line equations from the relay-end voltage and current (discussed in the next section). Below that is a table of impedance, rail current, leakage current and rail-to-rail voltage at each element.

- The lower-middle graphs plot rail current, rail-to-rail voltage and impedance (looking toward the relay end) as functions of distance. Minimum and maximum ordinate values are shown near the left side of each graph.

- The lower-right is a vector diagram showing the relative phases of feed voltage, track-coil voltage, track-coil current and local-coil current. Vector magnitudes are not to scale.

**Comparison With Known Results**

The default values of the tool, as shown in Figure 5, are based on the track circuit analyzed in [8], which includes a PTV-42 two-element vane relay with a pick-up of 0.122 V at 0.64 A. That analysis is a pencil-and-paper calculation using the transmission line equations (1) and (2). Calculations start with the track relay pick-up values and proceed toward the feed end, ending with the feed voltage, as in Figure 2.

In the subject tool, calculations start with the feed voltage and proceed toward the relay end, ending with the relay voltage and current. Voltage and current at each track element are calculated along the way, as in Figure 3. In addition to these forward calculations, the transmission line equations are applied here as well. They are used to work backward from the relay end to the feed end of the track as a check of the forward calculations and displayed in the lower-left text box of the tool.

In the screenshot of Figure 5, the feed voltage calculated in [8] was used as the starting point and fine-tuned to achieve the required track relay pick-up values. A comparison of key values between reference [8] and this tool are shown in Table 1.
In particular, note that

- The 100-element track circuit agrees well with the transmission line equations (compare \( E, I, e \) and \( i \)).

- There is a nearly 2° discrepancy between the two methods in the phase difference between the relay voltage and feed voltage. This is due to the different number of decimal places carried along in each method. Recalculating [8] by hand but using four decimal places results in the 84.7° phase difference found in the tool (which uses six decimal places internally). Alternatively, adjusting the tool’s software to use only three decimal places results in the 83° phase difference found in [8].

Similar favorable results are achieved modeling the track circuits of [5], [6] and [9].

### Number of Elements

Using the default track circuit of Figure 5, Table 2 compares the feed-end track voltage and track current of various numbers of elements in the quasi-distributed-leak model (\( E \) and \( I \), finite elements), including the center-leak method (one element), with the distributed-leak method (\( \bar{E} \) and \( \bar{I} \), \( \infty \) elements). For each different number of elements, the source voltage was adjusted to achieve the desired relay-end voltages and currents of Figure 5. Thus, \( \bar{E} \) and \( \bar{I} \) remain practically the same for all scenarios in Table 2 and are shown in the top section. The differences between \( \bar{E} \) and \( E \) and between \( \bar{I} \) and \( I \) increase as the number of elements decrease.

<table>
<thead>
<tr>
<th>Elements</th>
<th>( \bar{E} )</th>
<th>( \bar{I} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \infty )</td>
<td>0.527 V ( \angle 39.7° )</td>
<td>1.852 A ( \angle -14.8° )</td>
</tr>
<tr>
<td>Elements</td>
<td>( E_s )</td>
<td>( E )</td>
</tr>
<tr>
<td>----------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>100</td>
<td>1.889 V ( \angle -14.7° )</td>
<td>0.527 V ( \angle 39.8° )</td>
</tr>
<tr>
<td>50</td>
<td>1.888 V ( \angle -14.7° )</td>
<td>0.527 V ( \angle 39.8° )</td>
</tr>
<tr>
<td>20</td>
<td>1.888 V ( \angle -14.7° )</td>
<td>0.527 V ( \angle 39.8° )</td>
</tr>
<tr>
<td>10</td>
<td>1.887 V ( \angle -14.7° )</td>
<td>0.527 V ( \angle 39.9° )</td>
</tr>
<tr>
<td>5</td>
<td>1.886 V ( \angle -14.8° )</td>
<td>0.528 V ( \angle 40.0° )</td>
</tr>
<tr>
<td>2</td>
<td>1.879 V ( \angle -15.0° )</td>
<td>0.530 V ( \angle 40.3° )</td>
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<tr>
<td>1</td>
<td>1.862 V ( \angle -15.6° )</td>
<td>0.529 V ( \angle 40.4° )</td>
</tr>
</tbody>
</table>

Table 2—Effect of the number of elements.
APPLICATIONS

This tool can predict an approximate range of ballast conditions for a required shunting sensitivity. First, adjust the feed voltage for pick-up at minimum ballast resistance; then apply the shunt and start increasing the ballast resistance until the minimum drop-away values are reached. Using the default circuit values of Figure 5, the tool quickly shows that a 0.06-Ω shunt is effective through arbitrarily high ballast resistance, but there is some maximum value of ballast resistance at which a 0.25-Ω shunt, for example, may not be effective. Such a method may also indicate whether or not track circuit adjustments would be needed over the expected range of ballast conditions.

The tool presented here may also be used

- to predict shunting sensitivity: apply a shunt and increase its resistance until minimum drop-away values are reached, using maximum ballast resistance and the high end of the feed voltage regulation range.
- to approximate required settings of new or modified track circuits.
- for direct-current track circuits by setting the frequency to 0.
- as a training aid for signal engineers, designers and maintainers.
- for general transmission-line modeling. This tool has been used to verify leakage current due to capacitance of a long unterminated pair of wires, with the track portion representing the cable.

ENHANCEMENTS

There is much that can still be done to enhance the tool:

- Add feed- and relay-end transformers.
- Simulate broken rail and bypaths.
- Dynamically highlight the relay parameters to indicate their relation to pick-up, drop-away and ideal phase values.
- Specify regulation of feed voltage.
- Calculate feed voltage boost factor for non-ideal relay phase relations.
- Allow adjustment of local coil voltage.
- Allow disconnection of impedance bonds at both ends and all loads at the relay end, such as to demonstrate open-circuit ballast resistance measurement techniques (which can be approximated now by entering very large resistance values for these components).
- Support non-uniform ballast leakage.
- Create a scaled vector diagram of the complete circuit.

CONCLUSION

A software tool that analyzes certain kinds of track circuits was shown to provide results similar to other established methods of analysis. Calculations are performed from the feed end to the relay end so that the effects of changes in source voltage can be observed at the relay. The track portion of the circuit is modelled as a transmission line, as is usually the case, but with a finite number of transmission line elements. This allows rail-to-rail voltage, rail current and leakage current to be calculated at many points throughout the track, along with the effect of placing a test shunt anywhere along the track. A vector diagram is drawn and updated as changes are made, showing voltage and current relationships at the track relay.
REFERENCES


LIST OF FIGURES AND TABLES

Figure 1—Track (transmission-line) element.

Figure 2—Calculation flow using transmission-line equations (distributed leak).

Figure 3—Calculation flow using finite-element model (quasi-distributed leak).

Figure 4—Equivalent circuit of feed-end, track and relay-end components.

Figure 5—Screenshot of tool with default values.

Table 1—Comparison of reference and model circuit parameters.

Table 2—Effect of the number of elements.
Computer-Aided Analysis of Non-Coded Alternating-Current Track Circuits Using A Finite-Element Transmission-Line Model

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**Track Circuit**

- “Simple” circuit, reliably detects trains and component failures

**Lumped Elements**

- Lumped elements for rail impedance and ballast admittance
- "Center leak"
- Simple calculations (Ohm’s law, arithmetic)
- Not distributed

**Transmission Line Model**

- Developed for telegraph lines
  - Long parallel conductors
  - Series resistance and inductance
  - Shunt conductance and capacitance
- Repeated elements (distributed leakage)

**Track as Transmission Line**

- Rail impedance
  - R and L of model
- Ballast admittance
  - G and C of model
- Particular solution to the Telegrapher’s Equations
  - Feed end in terms of relay end using total Z and Y
Analysis with Transmission-Line Equations

- Use a solution to the Telegrapher's Equations, based on infinite elements
- "Distributed leak" with uniform track characteristics
- Popular method (in literature and spreadsheets)
- Calculations from relay to feed (arithmetic, hyperbolic trig, √)
- Black box – elements not exposed

Analysis with Finite-Element Model

- Between single element and infinite elements
- "Quasi-distributed leak"
- Calculations from feed to relay (Ohm's law, arithmetic)
- All elements exposed and individually calculated – track need not be uniform
- Similar calculations with or without shunt at any location

Track Circuit Model

- Arithmetic operations: series and parallel Z, Ohm's law, current dividers
- Admittance replaced with impedance

Comparison of TL and FE Methods

<table>
<thead>
<tr>
<th>Elements</th>
<th>Feed E (V)</th>
<th>Feed I (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.527, +29.8°</td>
<td>1.851, −14.8°</td>
</tr>
<tr>
<td>50</td>
<td>0.527, +29.8°</td>
<td>1.840, −14.9°</td>
</tr>
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<td>20</td>
<td>0.527, +29.8°</td>
<td>1.847, −15.0°</td>
</tr>
<tr>
<td>10</td>
<td>0.527, +29.9°</td>
<td>1.841, −15.2°</td>
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<td>5</td>
<td>0.526, +46.0°</td>
<td>1.832, −15.7°</td>
</tr>
<tr>
<td>2</td>
<td>0.529, +46.3°</td>
<td>1.802, −17.0°</td>
</tr>
<tr>
<td>1</td>
<td>0.529, +46.4°</td>
<td>1.754, −19.3°</td>
</tr>
</tbody>
</table>

2000 ft, Z = 0.1 Ω ∠60°, B = 2 D · kft

Unshunted Scenario

Shunted Scenario (0.06 Ω)
Shunted Scenario (0.5 Ω)

Applications
• Predict range of ballast conditions for required shunting sensitivity
  – Adjust feed voltage for pick-up at minimum ballast resistance
  – Apply shunt
  – Increase ballast resistance until minimum drop-away
• Predict shunting sensitivity
  – Apply shunt at maximum ballast resistance
  – Increase shunt resistance until minimum drop-away

Conclusion
• Compromise between a few lumped elements and infinitely-distributed elements
• Relay end as a function of feed end
• Expose V, I and Z throughout track
• Fast recalculation for “what if”