A Linear Programming Model for Optimization of the Railway Blocking Problem

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Abstract — We have developed a new network blocking optimization model that optimally assigns shipments to blocks and determines their subsequent routing plan. Results show that we can obtain optimal solutions (within 0.0114% of optimality) on sample data set from a Class I North American railway. In this paper, we describe this blocking optimization model, we present the integer linear programming (ILP) formulation, we show the computational results obtained, and discuss related future research.

Keywords — railway network design and optimization; blocking problem; railway operations; classification yards; handling costs; linear programming

I. INTRODUCTION

Railroads are one of the most efficient and one of the fastest methods for transporting freight in North America, and Class I railways ship millions of cars of goods and material every year [1]. Decisions on where and when to route various shipments can cost or save millions of dollars per year. Despite the fact that linear programming optimization methods have been applied to railway operations for decades [3], decisions on traffic classification and aggregation continue to be based primarily on empirical observation and experience rather than objective and provably optimal approaches. We will apply such techniques to produce optimal blocking plans (i.e., assignment of shipments to specific blocks of cars and subsequent routing of shipments via blocks to their destinations).

A. Background

A classification yard is a railway yard used to separate, sort and group cars according to their final destination(s), and also allows the inspection of trains [3-6]. A typical car can pass through several classification yards from its origin to its destination. However, cars are not reclassified at every yard that they visit; every time a car is reclassified, a lot of resources, time, and money are spent in this activity [3-6].

There are several types of classification yards, hump yards and flat yards being the most common [4]. Hump yards are characterized by a “hump” or hill, upon which the cars are pushed by an engine. At the top of the hill, cars are automatically decoupled and switched to the proper destination track, onto which, via gravity, they roll down and are now grouped with other cars heading on the same destination track. These yards are the most cost effective and with the largest capacity. In flat yards, the cars are moved onto the designated tracks by a locomotive, rather than via gravity assist [4].

For our purposes, we will consider a shipment to be made up of several whole cars with a common origin and destination, though in general, a shipment can be made up of a fractional car (and several shipments might be on a single car) [2-3,7]. Once shipments have left their origin and are in their way to their final destination, they can pass through numerous classification yards. In each of these classification yards, shipments may or may not be reclassified again, as befitting the specific circumstances. Every time a shipment is classified, the railway incurs additional costs due to human labor, the use of yard resources, time delays, etc. [2-3]. In order to avoid these extra...
costs, referred to here as *handling costs*, shipments that share a final destination (or shipments that have the possibility of traveling together for a portion of their journeys even though they may be destined to different final destinations) are grouped together to create a *block* [1-5]. A block is now paired with a new origin-destination set of yards, which may or may not correspond with the origin and destinations of any of the constituent shipments/cars. Once shipments are grouped together in a block, they get assigned to trains consisting of multiple blocks that share relevant portions of their routes. These shipments can then pass through a number classification yards with out being reclassified, only getting classified again when they reach the destination of the block. Once a block reaches its destination, it is disassembled and the constituent shipments/cars that are not at their own final destination are assembled into new blocks and continue on their way to their own final destinations.

**B. Blocking Problem**

The aim here is to develop a *blocking policy* or a *blocking plan* [6] that describes which blocks should be assembled at each classification yard, which shipments should be assigned to those blocks, and which series of blocks will carry each shipment to its final destination. Once a blocking policy is developed, the next logical and sequential step is to determine in which trains those blocks should travel. In order to do so, railways also develop train schedules and *block to train assignment* plans that are consistent with the blocking policy [1-2]. The blocking problem addressed herein focuses only on the assignment of shipments to blocks and the routing of blocks through the network. We will neither address the assignment blocks to specific trains, nor the movement or scheduling of those trains.

More precisely, we seek to develop blocking plans that minimize the total cost of routing all shipments in a railway network from their origins to their destinations while satisfying yard capacities and other operational constraints. Classification yards have two yard capacity limitations that are relevant here; *blocking capacity* and *shipment capacity* [8]. Blocking capacity is the maximum number of blocks that can be assembled at a yard, and is related to the number of classification tracks at the yard. Shipment capacity is the maximum number of cars that can be handled a yard, and is associated with the volume of passing traffic that a yard can manage. The specific values for these two capacities can vary quite considerably from yard to yard, primarily depending on the type and size of the yard in question; typically, a hump yard will permit larger blocking and shipment capacities, while flat yards will often have tighter limitations. Another important consideration is the *cars-to-block* requirement, which refers to the minimum number of cars that must be assigned to a block created at any particular yard. This is an operational constraint that typically arises from managerial decisions based on some perceived limit below which it is not thought to be worthwhile to assemble a block.

With the goal being to minimize the costs associated with the blocking plan, we need to consider two distinct costs. The first type of cost is the cost arising from the various blocks moving through the railway network; these costs are called *distance traveled costs*, and are generally proportional to the lengths of the track they have been routed over, though there may be other considerations as well (e.g., an uphill section of track may have a higher distance travelled cost than a downhill section of track of the same length). The second type of cost is the cost related to intermediate handlings or reclassification of shipments at the classification yards; these costs are called *handling costs*. In general, handling costs can vary throughout the network; each classification yard might have its own handling cost, depending on the size and type of yard. Hump yards typically have lower handling costs and flat yards typically have higher handling costs.

**C. Goals and Motivation**

Current practice in developing a blocking plan is for an experienced expert to manually assign shipments to blocks using his best judgment. In the case of Class I North American railways, the network is typically broken down into smaller more manageable sections, and each of these is “optimized” individually before those individual plans are rationalized into a single all-inclusive blocking plan. This process is carried out roughly weekly, when the shipments for the coming days are firmed up. And although there may be significant changes in shipments from week to week, the blocking plan for any given week will often be heavily drawn from plans of the previous week(s) due to the difficulty in manually producing a new blocking plan from scratch. This approach is commonplace across the industry, but there is often no real understanding of how good a particular blocking plan may be, and whether significant savings can be realized with a more optimal plan. Our goal herein is to develop an *integer linear programming* (ILP) design model that will optimize a blocking plan given a specified set of shipments [9].
II. **ILP Design Model for the Blocking Problem**

Our ILP design model makes use of the following notation:

Sets:
- $K$ is the set of shipments.
- $Y$ is the set of yards in the rail network.

Decisions variables:
- $x_{ijk} \in \{0,1\}$ denotes whether shipment $k$ is assigned to a block between yards $i$ and $j$ ($x_{ijk} = 1$ if it is, and $x_{ijk} = 0$ if it is not).
- $y_{ijk} \in \{0,1\}$ denotes whether the block between yards $i$ and $j$ is an intermediate block for shipment $k$ ($y_{ijk} = 1$ if it is, and $y_{ijk} = 0$ if it is not).
- $z_{ij} \in \{0,1\}$ denotes whether a block between yards $i$ and $j$ is created ($z_{ij} = 1$ if such a block is created, and $z_{ij} = 0$ if it is not).

Input parameters:
- $s_k$ is the number of cars in shipment $k$.
- $o_k$ is the origin yard of shipment $k$.
- $d_k$ is the destination yard of shipment $k$.
- $h_i$ is the handling cost per car at yard $i$.
- $c_{ij}$ is the distance traveled cost per car in a block between yards $i$ and $j$.
- $m_i$ is the minimum number of cars permitted in a block created at yard $i$.
- $B_i$ is the maximum number of blocks that can be created at yard $i$.
- $C_i$ is the maximum total number of cars that can be handled at yard $i$.

Using the above notation, we can construct an integer linear programming formulation that seeks to minimize total costs incurred in assigning all shipments to blocks in a manner that permits their routing from origin to destination while meeting capacity and other limitations. Our ILP is as follows.

**Minimize:**

$$
\sum_{k \in K} \sum_{(i,j) \in \mathcal{P}} c_{ij} x_{ijk} + \sum_{k \in K} \sum_{(i,j) \in \mathcal{P}} h_i y_{ijk} s_k
$$

**Subject to:**

$$
\sum_{j \in Y} x_{ijk} - \sum_{j \in Y} x_{jik} = \begin{cases} 
1, & \text{if } i = o_k \\
0, & \text{if } i \neq o_k \land i \neq d_k \\
-1, & \text{if } i = d_k 
\end{cases} \\
\forall k \in K
$$

(2)

If $\sum_{k \in K} x_{ijk} \geq 0$, then $\sum_{k \in K} x_{ijk} s_k \geq m_i$ \hspace{1cm} $\forall i \in Y, \forall j \in Y$ 

(3)

$$
\sum_{j \in Y} x_{ij} \leq B_i \\
\forall i \in Y
$$

(4)

$$
\sum_{k \in K} \sum_{i \in Y} x_{ijk} s_k \leq C_i \\
\forall j \in Y
$$

(5)

$$
x_{ijk} = y_{ijk} \\
\forall i \neq o_k, \forall j \in Y, \forall k \in K
$$

(6)

$$
y_{ijk} = 0 \\
\forall i = o_k, \forall j \in Y, \forall k \in K
$$

(7)

$$
z_{ij} \geq x_{ijk} \\
\forall i \in Y, \forall j \in Y, \forall k \in K
$$

(8)

$$
\sum_{(i,j) \in \mathcal{P}} x_{ijk} \geq 1 \\
\forall k \in K
$$

(9)

$$
x_{ilk} = 0 \\
\forall k \in K, \forall i \in Y, \forall j \in Y: i = j
$$

(10)
\begin{align*}
\sum_{i \in Y} x_{ijk} & \leq 1 \quad \forall k \in K, \forall j \in Y 
\sum_{j \in Y} x_{ijk} & \leq 1 \quad \forall k \in K, \forall i \in Y 
x_{ijk} + x_{jik} & \leq 1 \quad \forall i \in Y, \forall j \in Y, \forall k \in K
\end{align*}

The objective function in equation (1) is the sum of all handling and distance travelled costs incurred for each shipment as assigned to and routed within each block. The constraints in equation (2) ensure that each shipment is assigned to a single block leaving its origin yard and a single block arriving at its destination yard, and that flow conservation is maintained at all intermediate yards (i.e., if a shipment arrives at a yard that is not its destination, it will leave that yard on another block). The constraints in equation (3) ensure that if a block is created, it will consist of some specified minimum number of cars. As written above, this constraint is non-linear, but it is easily linearized by the introduction of a dummy variable, \( w_{ij} \), and transforming equation (3) into equations (14) and (15) below (where \( L \) is an arbitrarily large number).

\begin{align*}
m_i - \sum_{k \in K} x_{ijk}s_k & \leq L \cdot w_{ij} \quad \forall (i, j) \in P \quad (14) 
\sum_{k \in K} x_{ijk} & \leq L \cdot (1 - w_{ij}) \quad \forall (i, j) \in P \quad (15)
\end{align*}

The constraints in equation (4) satisfy the blocking capacity constraint by limiting the number of blocks created at a yard. Similarly, constraints in equation (5) guarantee the fulfillment of the shipment capacity constraint by restricting the amount of traffic at a yard. Equations (6) and (7), together, determine if there are intermediate handlings, or not, when assigning each shipment to a block. Equation (8) determines whether a block between any two yards is created, or not. Constraints in equation (9) assign all shipments to at least one block, making them move through the railway network. Constraints in equation (10) prohibit the creation of blocks with the same yard as origin and destination. Equations (11) and (12) disallows redundant traveling of shipments, guaranteeing that once a shipment has entered or left a yard, respectively, this shipment will not return to the same yard. Constraints in equation (13) prevent sub-tours of a shipment between two yards. We observed that without these constraints, in rare cases, the model assigned a block to a nicely behaved simple route from origin to destination, but also assigned it to a separate phantom route over a set of three or four rail segments looping back onto itself. It did so because in those cases in order to meet the minimum number of blocks required to be created at a yard, and it was less costly to pay for a phantom sub-tour than to actually route some blocks through those yards.

### III. Experimental Study

Our ILP formulation was implemented in AMPL, and was solved using Gurobi Optimizer 5.6.3 on a Dell PowerEdge R420 server with a 2.2 GHz processor, and 128 GB of RAM. “Fully optimal” (i.e., not early terminated) results are based on full Gurobi terminations with an optimality gap setting of 0.0001, meaning solutions are guaranteed to be within 0.01% of optimal.

The ILP was tested on two benchmark networks. Network #1, shown in Figure 1 is a trivially small five-node network created to clearly define, exemplify, and validate the problem. Each node represents an active classification yard. This network can be thought of as a small local railway with only five yards, but it might also represent a larger railway consisting of many small yards (not shown) whose traffic is all routed through the five larger classification yards shown (i.e., local and regional traffic of small flat yards have already been grouped into blocks to reach the major classification yards as their first destinations). We can note here that this latter representation is not an uncommon practice; a larger rail network is often reduced to high-level representation consisting of only the major classification yards. An initial sub-problem is solved (generally in an ad hoc manner) to determine which major classification yards each shipment should be initially routed to and ultimately destined to at the other end; the simplest approach would be to identify the nearest major classification yards to the shipment’s origin and destination, and assign them to blocks to/from those major yards. For our purposes, however, we considered the rail network to consist of only the five yards shown.

Network #1 consisted of 70 shipments that needed to be routed from their respective origins to their respective destinations. All five classification yards were allowed be the origins and/or destinations of the various shipments, and were allowed to be the origins and/or destinations of blocks. Distance traveled costs are proportional to the
distances between the yards as drawn in the schematic, and ranged from 1 for the shortest rail segment to 8 for the longest rail segment (cost units are arbitrary). Since the initial classification of a shipment at its origin is unavoidable and constant, handling costs were only considered in the event of reclassification (i.e., intermediate handling), and these costs were 5 for all yards. For this network, values for the maximum numbers of blocks per yard, and maximum numbers of shipments at a yard were set so large that the routing was not limited by these two constraints. The minimum number of cars per block was set to 5 cars.

Figure 1. Network #1 Schematic Representation.

Network #2, shown in Figure 2, is an isomorphic representation of a regional section of a Class I North American railway. Network #2 consists of 39 nodes representing the classification yards of that regional network, and there were 1,052 shipments (allocated in 4,443 cars) that required routing. As with Network #1, all classification yards were allowed to be the origins and/or destinations of the various shipments, and were allowed to be the origins and/or destinations of blocks. Distance traveled costs are proportional to the distances between the yards as drawn in the schematic, and ranged from 10 for the shortest rail segment to 3,377 units for the longest rail segment. Since the initial classification of a shipment at its origin is unavoidable and constant, handling costs were only considered in the event of reclassification (i.e., intermediate handling), and these costs were ranged between 10 to 40 units, depending on the yard. Other parameters were set as follows: the maximum number of blocks per yard was 100, the maximum number of cars at a yard ranged between 50 and 1,000 (dependent on the size of the yard in question), and minimum number of cars per block was 5.

Figure 2. Network #2 Schematic Representation.
IV. RESULTS AND DISCUSSION

The benchmark solution for the Network #1 blocking problem was obtained as an exercise performed by a group of ten railway industry experts at an optimization workshop. Each was given 45 minutes to devise the best blocking plan they could. As shown in Table 1, their best solutions on average had a “cost” of 1.1254, as normalized to the cost of the optimal ILP solution (costs were normalized to be independent of the scale of the railway networks tested and for ease of comparison). Subsequently the ILP formulation above was used to solve the same problem and required a little less than one second to arrive at the optimal solution with a “cost” of 1.000.

<table>
<thead>
<tr>
<th>NETWORK</th>
<th>BENCHMARK</th>
<th>MILP FORMULATION</th>
<th>NOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time</td>
<td>Cost</td>
<td>Time</td>
</tr>
<tr>
<td>#1</td>
<td>2700 sec</td>
<td>1.1254</td>
<td>1 sec</td>
</tr>
<tr>
<td>#2</td>
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<td>0.5 hr</td>
<td>3.2321</td>
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<tr>
<td></td>
<td></td>
<td>2 hrs</td>
<td>2.4791</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12 hrs</td>
<td>1.6287</td>
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<tr>
<td></td>
<td>46.5 hrs</td>
<td>6.700</td>
<td>286 hrs</td>
</tr>
</tbody>
</table>

The benchmark solution for Network #2 was obtained by one of the authors, who received training from an industry expert; that expert develops weekly blocking plans for a Class I North American railway. The benchmark solution took approximately 46.5 hours to complete (though we understand that the typical industry expert would complete this in under 1-2 business days), but had a cost that was 670% higher than the optimal solution obtained via the ILP formulation. In other words, the optimal solution provides an 85% reduction in total costs associated with the blocking plan. However, as can be seen in Table 1, the optimal ILP solution required 286 hours (nearly 12 days) to complete, which would not be practical for obtaining weekly blocking plans. As such, we reran our ILP formulation on Network #2 but terminated it early (i.e., before reaching a strictly optimal solution) to determine whether sub-optimal but still good solutions could be obtained in reasonable time. In three such test cases where we terminated the ILP after 30 minutes, 2 hours, and 12 hours of runtime, we obtained solutions with costs that were 3.2321, 2.4791, and 1.6287 times larger than the optimal, respectively. While this may appear to characterize quite poor performance, these time-limited solutions represent 51.76%, 62.99%, and 75.69% reductions in total blocking-related costs when compared with the benchmark.

Although the time spent in obtaining the optimal solution for Network #2 does not sound particularly inspiring at first, there are few aspects to consider while overviewing the advantages of the ILP formulation. The fact that the ILP model can provide a strictly optimal solution to the problem is a great advantage to the railway industry, which often works with suboptimal solutions. Since blocking decisions often need to be made in a short period of time (say, one, or at most a few, business day(s)), waiting 10+ days would generally be unreasonable, even for a strictly optimal blocking plan. However, the ILP model can still offer a number of advantages to an operator.

First, as noted above, a time-limited run of the ILP can still produce solutions providing significant savings relative to the benchmark approach. In the case where the operator was willing to wait as much 12 hours (a favorable runtime relative to the typical industry expert’s benchmark), the ILP provides cost reduction of nearly 76% relative to the benchmark. If the operator requires a much more rapid solution and is willing to sacrifice optimality, a 30 minute solution of the ILP will still provide a nearly 52% cost reduction relative to the benchmark (while it is virtually impossible for an industry expert to obtain any solution in such a short period of time).

Second, since a large amount of railway traffic recurs weekly (or seasonally or at some other regular intervals) and a large number of shipments will be common from one week to the next, the ILP can be run to optimality to obtain a good long-term blocking plan, which can be tweaked weekly as shipments vary. The ILP can be rerun every few weeks when the shipments have varied beyond some specified degree. It is also possible to periodically run a variant of the ILP where an existing blocking plan can be used as an initial solution, and only new or modified shipments are allowed to be modified. This would be accomplished by splitting the shipments into two sets (say, \( K_{exist} \) and \( K_{new} \)), and introducing a new set of constraints that sets all variables involving \( K_{exist} \) to specified values. Our experience suggests that such a sub-problem will solve significantly faster than the full problem, so it is possible that this periodic incremental blocking plan could be solved to optimality or near-optimality quickly enough to be useful.
V.  CONCLUSIONS

We have developed an ILP blocking optimization model that provides optimal solutions to the railway blocking problem, which can lead to significant savings in related operational costs (handling and distance travelled costs). In the main test case network solved herein, derived from a regional section of a Class I North American railway network, the optimal solution obtained was 85% less costly than the benchmark solution. Furthermore, although the strictly optimal solution takes a considerable amount of time to obtain (nearly 12 days in our main test case), the ILP can also be used to obtain sub-optimal solutions in reasonable time that still provide significant savings over the benchmark solution.

An attractive option for future work will be to investigate the use of heuristic approaches and decomposition methods to explore the possibility of reducing runtime for near-optimal solutions and/or for developing blocking plan for particularly large railway networks. Another logical step for further work is to develop a block to train assignment model that will not only determine the blocking plan but also assign those blocks to specific trains.

VI.  REFERENCES


A Linear Programming Model for the Optimization of the Railway Blocking Problem

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Optimization of the Railway Blocking Problem

A Linear Programming Model for the
Outline
• Introduction
• Background
• Blocking Problem
• Goals and Motivation
• ILP Design Model
• Experimental Approach
• Results and Analysis
• Concluding Discussion and Future Work

Introduction
• Railroads are arguably the most efficient and fastest methods for transporting freight within North America
• Major North American railway companies ship millions of cars over their railroads every year
• Decisions on where and when to route various shipments can cost or save millions of dollars per year
• Traffic classification and aggregation is still based primarily on empirical observation and experience
• Opportunity for objective and provably optimal approaches

Outline
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• Experimental Approach
• Results and Analysis
• Concluding Discussion and Future Work

Background
• Classification yards separate, sort, and group cars
• A shipment can be made up of several whole cars or fractions of car
• A typical shipment or car can pass through several classification yards from its origin to its destination
• Every time a shipment is classified, the railway incurs additional costs
• Shipments are grouped together to create a block
• After a shipment is placed in a block, it is not reclassified until it reaches the destination of that block, reducing intermediate handlings of shipments

Blocking Problem
• A blocking plan describes which blocks should be assembled at each classification yard, which shipments should be assigned to those blocks, and which series of blocks will carry each shipment to its final destination
  – Subsequently includes development of train schedules and block to train assignment plans that are consistent with the blocking plan
• A blocking plan that produces the minimal total shipment cost can lead to huge savings in handling costs for the railway industry
Blocking Limitations and Parameters

• Yard capacity
  – Blocking capacity
  – Shipment (car) capacity

• Operational constraints
  – Cars-to-block requirement

• Costs
  – Distance traveled costs
  – Handling costs

Outline

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Goals and Motivation

• Current practices
  – Expert judgment
  – Lack of tools
  – Manual assignments
  – Small sections

• Main goal:
  – Develop an integer linear programming (ILP) design model that will optimize a blocking plan given a specified set of shipments

Reminder: Linear Programming Model

• Terminology:
  – Decision variables are quantities you can control, which completely describe the set of decisions to be made.
  – Constraints are limitations on the values of the decision variables.
  – The objective function is a measure that can be used to rank alternative solutions (e.g., NPV, cost, production rate, travel time).
  – The goal is to either maximize or minimize its value.
  – A solution is any combination of values for all decision variables.
  – A feasible solution is a solution that satisfies all of the constraints.
  – An infeasible solution doesn’t satisfy some constraint(s).
  – An optimal solution is the best feasible solution.

Reminder: Linear Programming Model (2)

• Linear Programs (LP): the objective and constraint functions are linear and the decision variables are continuous.
• Integer Linear Programs (ILP): one or more of the decision variables are restricted to integer values only and the functions are linear.
• Pure IP: all decision variables are integer.
• Mixed IP (MIP): some decision variables are integer, others are continuous.
• 1/0 MIP: some or all decision variables are further restricted to be valued either “1” or “0”.

Reminder: Linear Programming Model (3)

• General symbolic form

Objective Function

Minimize: \[ c_1 x_1 + c_2 x_2 + \cdots + c_n x_n \]  

Subject to: \[ a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \text{ (} \leq \text{ or } = \text{) } b_1 \]
[ ]
[ ]
[ ]

Bounds

\[ x_j \geq 0, \quad \text{for } j = 1, \ldots, n \]

...where \( a \), \( b \), and \( c \) are the model parameters
Outline

- Introduction
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ILP for Blocking Problem Notation

- Sets:
  - \( K \) is the set of shipments
  - \( Y \) is the set of yards in the rail network
- Decisions variables:
  - \( x_{ijk} \in \{0,1\} \) denotes whether shipment \( k \) is assigned to a block between yards \( i \) and \( j \) \( (x_{ijk} = 1 \) if it is, and \( x_{ijk} = 0 \) if it is not).
  - \( y_{ijk} \in \{0,1\} \) denotes whether the block between yards \( i \) and \( j \) is an intermediate block for shipment \( k \) \( (y_{ijk} = 1 \) if it is, and \( y_{ijk} = 0 \) if it is not).
  - \( z_{ij} \in \{0,1\} \) denotes whether a block between yards \( i \) and \( j \) is created \( (z_{ij} = 1 \) if such a block is created, and \( z_{ij} = 0 \) if it is not).

ILP for Blocking Problem

\[
\begin{align*}
\min & \quad \sum_{k \in K} \sum_{i,j \in Y} c_{ijk} x_{ijk} \\
\text{subject to} & \quad \sum_{j \in Y} x_{ijk} \leq B_i \\
& \quad \sum_{i,j \in Y} x_{ijk} y_{ijk} \leq C_j \\
& \quad y_{ijk} = 0 \\
& \quad z_{ij} \geq x_{ijk} \\
& \quad \forall i \in Y, \forall j \in Y, \forall k \in K 
\end{align*}
\]

ILP for Blocking Problem (2)

\[
\begin{align*}
\sum_{i,j \in Y} z_{ij} & \leq B_i \\
\sum_{k \in K} \sum_{i,j \in Y} x_{ijk} y_{ijk} & \leq C_j \\
y_{ijk} & = y_{ijk} \\
z_{ij} & \geq x_{ijk} \\
\forall i \in Y, \forall j \in Y, \forall k \in K
\end{align*}
\]

ILP for Blocking Problem (3)

\[
\begin{align*}
\sum_{i,j \in Y} x_{ijk} & \geq 1 \\
x_{ijk} & = 0 \\
y_{ijk} & \leq 1 \\
z_{ij} & \leq 1 \\
\forall k \in K, \forall i \in Y, \forall j \in Y, \forall k \in K
\end{align*}
\]
We tested the ILP on two benchmark networks.

- Benchmark solutions were obtained for both networks:
  - Industry experts
  - Author trained by industry expert

The ILP was implemented in AMPL, and solved using Gurobi Optimizer 5.6.3 on a Dell PowerEdge R420 server with a 2.2 GHz processor, and 128 GB of RAM.

"Fully optimal" (i.e., not early terminated)
- Optimality gap setting of 0.0001 (solutions are guaranteed to be within 0.01% of optimal)

Network #1
- Network #1 is a trivially small five-node network
  - 5 classification yards, serving as shipment origins and destinations
  - 70 shipments that need to be routed

\[
\begin{align*}
h &= 5 \\
c_i &\text{ranged from 1 to 8} \\
m &= 5 \\
B &\text{arbitrarily large number} \\
C_i &\text{arbitrarily large number}
\end{align*}
\]

Network #2
- Isomorphic representation of a regional section
  - 39 classification yards
  - 4,443 cars consisting of 1,053 shipments

\[
\begin{align*}
h &\text{ranged from 10 to 337} \\
c_i &\text{ranged from 10 to 40} \\
m &= 5 \\
B &= 100 \\
C_i &\text{ranged from 50 to 1,000}
\end{align*}
\]
Results (2)

- Results show that we can obtain optimal solutions (within 0.0114% of optimality) on sample data set from a Class I North American railway

<table>
<thead>
<tr>
<th>NETWORK</th>
<th>START-VARDD</th>
<th>END-VAIRED</th>
<th>NOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>0.0001 sec.</td>
<td>1.0001 sec</td>
<td>0.8292</td>
</tr>
<tr>
<td>#2</td>
<td>46.5 hrs</td>
<td>1.0001 sec</td>
<td>0.4824</td>
</tr>
<tr>
<td></td>
<td>2 hr</td>
<td>0.3700</td>
<td>Early Termination (59.7% migaug)</td>
</tr>
<tr>
<td></td>
<td>32 hr</td>
<td>0.2431</td>
<td>Early Termination (38.6% migaug)</td>
</tr>
<tr>
<td></td>
<td>386 hrs</td>
<td>0.1492</td>
<td>Fully Optimal</td>
</tr>
</tbody>
</table>

Concluding Discussion

- We have developed an ILP model that provides optimal solutions to the railway blocking problem
  - These solutions can lead to significant savings in related operational costs
- Our resulting blocking plans are more cost effective than those resulting from the benchmarks
  - Optimal solution obtained for railway network #2 was 85% less costly than the benchmark
  - The ILP can also be used to obtain sub-optimal solutions in reasonable time

Future Work

- We will investigate the use of heuristic approaches and decomposition methods to
  - explore the possibility of reducing runtime for near-optimal solutions
  - develop blocking plans for particularly large railway networks
- We will also develop a block to train assignment model that will not only determine the blocking plan but also assign those blocks to specific trains.

Appendix

- Sub-tour
  - Eliminating sub-tours is very challenging